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ABSTRACT

The Race of Man and Machine: Implications of Technology When Abilities and Demand Constraints Matter

In "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment," Acemoglu and Restrepo (2018b) combine the task-based model of the labor market with an endogenous growth model to model the economic consequences of artificial intelligence (AI). This paper provides an alternative endogenous growth model that addresses two shortcomings of their model. First, we replace the assumption of a representative household with the premise of two groups of households with different preferences. This allows our model to be demand constrained and able to model the consequences of higher income inequality due to AI. Second, we model AI as providing abilities, arguing that 'abilities' better characterises the nature of the services that AI provide, rather than tasks or skills. The dynamics of the model regarding the impact of AI on jobs, inequality, wages, labor productivity and long-run GDP growth are explored.

JEL Classification: O47, O33, J24, E21, E25

Keywords: technology, artificial intelligence, productivity, labor demand, income distribution, growth theory

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1 Introduction

In their paper "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment" Acemoglu and Restrepo (2018b) combine "task-based models of the labor market with directed technological change models" [Ibid, p.1492). They use their model, henceforth the AR-model, to examine the impact of automation technologies, such as Artificial Intelligence (AI),[1] on employment, growth and inequality.

While a significant contribution to the literature, there are two shortcomings in the AR-model. The first is that its reinstatement effects (the creation of new tasks and jobs due to the productivity gains enabled by AI) will depend, over the long-run, on the impact of AI automation on income distribution. If income inequality worsens, such as that the labor share in GDP declines, aggregate demand will decline. This would reduce the economy's actual and potential growth. Lower growth in turn would limit the reinstatement of new jobs.

The AR-model cannot take this into account, as it is supply-driven and hence lacks a mechanism to take into account the consequences of an increase in income inequality, as in all supply-driven growth models (Dutt, 2006). In a related paper, Acemoglu and Restrepo (2019a, p.228) recognizes this shortcoming.[2] In the present paper, drawing on Gries (2020b) and Gries and Naudé (2020) we address this shortcoming by allowing for growth in our model to be demand constrained by replacing the typical assumption of a representative household by the assumption of two groups of households with different preferences.

The second shortcoming shortcoming of the AR-model, which is due to the task-approach to labor markets on which it is based (see e.g. Autor et al. (2003); Autor and Dorn (2013)), is that it inadequately engages with the nature of AI and its technological feasibility. The task-

[1] AI is the "most discussed automation technology' (Acemoglu and Restrepo, 2018b, p.2).

[2] Other scholars who have identified aggregate demand as a crucial determinant of the effect of automation on jobs and growth include Bessen (2018) and Benzell et al. (2018) and Sachs et al. (2015).

approach is concerned with tasks and skills but not with *abilities*, although abilities better characterises the nature of the services that AI provide (Hernández-Orallo, 2017). According to Tolan et al. (2020, p.6-7) abilities are "a better parameter to evaluate progress in AI" because AI provide abilities to do tasks, and not skills, which are a human attribute requiring experience and knowledge, and not an attribute of AI. In the present paper, drawing on Gries and Naudé (2021), we incorporate AI as providing abilities.

The rest of the paper will proceed as follows. In section 2 an (semi) endogenous growth model is introduced that includes constraints from the demand-side, and that modifies the naive task-approach to labor markets. In section 3 the model is solved, and in section 4 the dynamics of the model in terms of the impact of AI on jobs, inequality, wages, labor productivity and long-run GDP growth are explored. Section 5 considers the impact of AI with simultaneous demand shocks. Section 6 concludes.

Our paper contributes to the recent theoretical literature on AI and economic growth modelling, such as the AR model, but also work by Aghion et al. (2017), Cords and Prettner (2019), Hémous and Olsen (2018) and Prettner and Strulik (2017). Unlike these models, the model presented here incorporate demand-constraints and a modified task-approach to labor markets.

2 A New Theoretical Model: Labor Tasks, Demand, and Growth

We start off (in 2.1) by describing the production of final consumption goods by sales-maximising firms who use labor, intermediate goods, as well as Artificial Intelligence (AI). In section 2.2 the nature of the relationship between AI and labor is set out, in 2.3 intermediate goods production is specified, and in section 2.4 aggregate budget constraints, income, and its distribution are derived. After having dealt with aggregate supply, we then focus our attention on aggregate demand in section 2.5. Here we introduce a novelty of this paper, namely the substitution of the typical assumption in endogenous growth models of a representative household by the assumption of two groups of households with different preferences.

2.1 Final goods-producing firms

Final goods for consumption are produced by firms using labor, AI services, and intermediate inputs. Actual sales of output may fall short of potential sales due to market frictions and shocks in final goods markets. To maximize sales, firms will incur marketing and product placement activities, buy labor and AI services in a competitive market, and purchase intermediate goods. The following sub-sections elaborate this maximization problem.

2.1.1 Output of Final Goods

Let firm $i \in \mathcal{F}$ be a representative firm that produces final goods using labor, AI services and intermediate inputs. The combination of labor and AI provide what we term ***human services***. We combine labor and AI into human services because AI is a software and information technology that is human-related in that it provides abilities to produce goods, but requires the skills and experience and knowledge of humans to add value. Skills and experiences are not in the domain of AI (Tolan et al., 2020). We denote human service inputs by H_{Qi}.

In addition to human services, the firm sources $N_i(t)$ differentiated intermediate inputs $x_{ji}(t)$ which are offered by $N(t)$ intermediate input-producing firms.

Given human service inputs and intermediate inputs, $Q_i(t)$, the potential output of final goods by firm i, is

$$Q_i(t) = H_{Qi}^{1-\alpha} \sum_{j=1}^{N_i} x_{ij}^{\alpha}(t) = N_i(t) H_{Qi}^{1-\alpha} x_i^{\alpha}(t) \tag{1}$$

2.1.2 Market frictions, sales promotion, and expected sales

Due to stochastic market frictions, not all of a firm's potential output will be sold. Assume that firm i can only sell Φ so that its effective sales ratio is $\phi_i(t) = \frac{\Phi_i(t)}{Q_i(t)} \leq 1$. The firm's subjective interpretation of $\phi_i(t) \leq 1$ is that this shortfall is due to the fact that customers are insufficiently informed about products, prices, qualities, and general market conditions. The extent of this mismatch[3] between potential and actual sales, $\delta_i(t)$ determines the effective sales ratio, i.e.:

$$\phi_i(t) = 1 - \delta_i(t) \tag{2}$$

As a response to a sub-optimal effective sales ratio, firms allocate human services $H_{\phi i}$ to promote sales so as to counter this mismatch (δ_i) and improve the likelihood of selling all potential output in the market. The match-improving mechanism can be formulated as $m_i = m_i(H_{\phi i})$, with $\frac{\partial m_i(t)}{\partial H_{\phi i}(t)} > 0$. Given that δ_i' denotes the stochastic market frictions which the firm perceives as exogenous, the total mismatch of potential and actual sales is

$$\delta_i(t) = \delta_i'(t) - m_i(H_{\phi i})$$

Each individual firm i observes that the expected effective sales ratio $E[\phi_i]$ is monotonically increasing with $H_{\phi i}$, and decreasing with δ_i',such that

$$E[\phi_i] = E[\phi_i(\delta_i', H_{\phi i})] \quad \text{with} \quad \frac{\partial E[\phi_i]}{\partial H_{\phi i}} > 0, \quad \frac{\partial E[\phi_i]}{\partial \delta_i'} > 0. \tag{3}$$

[3]The matching model with frictions that we draw on here is closely related to that of Gries (2020a).

2.1.3 Factor demands

Having described the firm's output and expected effective sales ratio in the previous two sub-sections, we can now derive the firm's factor demands from its profit maximization. We start by denoting the price of the human services factor as p_H, and the price of intermediate inputs x as p_x. The firm's profit maximization function then is:

$$\max_{H_{Qi},H_{\phi i},x_i} : E\left[\Pi_i\left(t\right)\right] = E\left[\phi_i\left(t\right)\right] Q_i\left(t\right) - p'_H\left(t\right)\left(H_{\phi i}\left(t\right) + H_{Qi}\left(t\right)\right) - N_i\left(t\right) p_x\left(t\right) x_i\left(t\right) \tag{4}$$

To maximize profits, firms first have to to organize an efficient sales process, and secondly, they need to determine optimal production.

First, consider the organization of an *efficient sales process.* Firm i allocates $H_{\phi i}$ to the search and information process and improves its effective sales. In order to sell all potential output, the firm increases $H_{\phi i}$ until all goods that have been produced and supplied can be expected to be absorbed by the market. The firm's total revenues $E\left[\phi_i\right] Q_i$ are determined by the expected success rate of selling the produced output $E\left[\phi_i\right]$ and the production of even more goods Q_i. As each element depends on the respective human service input, we assume that placing an already existing (but not yet demanded) output in the market is more effective than producing a new unit of output. That is, until the point when all production in fact finds a customer, the marginal revenue generated by human services in the matching process is greater than the marginal revenue of human service in production, and zero otherwise

$$\frac{\partial E\left[\phi_i\right]}{\partial H_{\phi i}} Q_i > E\left[\phi_i\right](1-\alpha)\frac{Q_i}{H_{Qi}} \quad for \quad E\left[\phi_i\right] \leq 1.$$

As a result, the firm will increase $H_{\phi i}$ until the expected sales ratio becomes

$$E\left[\phi_i\right] = 1, \tag{5}$$

and thus no unsold output remains. The firm will be in a sales equilibrium. Any time when conditions (5) holds, (3) defines a function for the allocation of labor to each firm i's sales activities[4]

$$H^*_{\phi i} = H_{\phi i}\left(E\left[\delta'_i\right]\right), \quad \frac{\partial H_{\phi i}}{\partial E\left[\delta'_i\right]} > 0 \tag{6}$$

Secondly, the firm needs to determine *optimal factor inputs*. Under the condition that $E\left[\phi_i\right] = 1$, a firm's profit (4) is

$$E\left[\Pi_{Qi}\right] = Q_i - N_i p_x x_i - p'_H\left(H^*_{\phi i} + H_{Qi}\right)$$

As p'_H is the price payable to human services (also in production), the first-order condition for the efficient use of labor in production gives

$$H_{Qi}\left(t\right) = \left(1-\alpha\right) Q_i\left(t\right) p'_H\left(t\right)^{-1} \tag{7}$$

and the demand for intermediate goods can be derived as

$$x_i\left(t\right) = \left(\frac{\alpha}{p_x\left(t\right)}\right)^{\frac{1}{1-\alpha}} H_{Qi} \tag{8}$$

2.2 Human services: Labor and AI

In this section we elaborate the human services input, and clarify the relationship between labor and AI. We draw on Gries and Naudé (2021).

[4]See appendix A for the Implicit Function Theorem.

2.2.1 The production of human services

Human services H_i, as already indicated, consists of labor and AI. It is produced following the task-based approach. As such, $H = H(L_L, A_L, A_{IT}, L_{IT})$, where L_L is the number of workers each providing one unit human experience, A_L is an index of general knowledge, A_{IT} is the total number of Machine Learning (ML) abilities (e.g. software algorithms) in the economy, and L_{IT} the IT-labor providing IT skills. Hence, our approach enriches and extends the naive task-approach by integrating human skills and experience with AI abilities, as per the arguments of Hernández-Orallo (2017) and Tolan et al. (2020). Furthermore, L_L and L_{IT} are different groups of labor, allowing us to have two separated segments in the labor market. The general function $H = H(...)$ can be specified as

$$H = \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} \tag{9}$$

where z denotes each task in a unit interval $[N-1, N]$, and $h(z)$ is the output of task z. As tasks range between $N-1$ and N, the total number of tasks is constant. While formally following the task-based approach, the more explicit specification of the nature of AI and its technological feasibility (reflected in A_{IT}) is a novel contribution.

Each task can either be produced only with labor, $l(z)$, or only with AI-labor services, $l_{IT}(z)$, if the task can be automated. Therefore, there are two sets of tasks. Tasks $z \in [N-1, N_{IT}]$ can be produced by both labor and AI services, and tasks $z \in (N_{IT}, N]$ can only be produced by labor. Thus, the output of a task can be generated in two ways, namely

$$h(z) = \begin{cases} A_L \gamma_L(z) l(z) + A_{IT} \gamma_{IT}(z) l_{IT}(z) & \text{if } z \in [N-1, N_{IT}] \\ A_L \gamma_L(z) l(z) & \text{if } z \in (N_{IT}, N] \end{cases} \tag{10}$$

Here $\gamma_L(x)$ is the classic productivity of labor of task z and A_L generally available knowledge, which is usable without rivalry and labor augmenting.

The AI service consists of three elements which reflects the fact that modern AI affects the workplace through the combination and interaction between skills, knowledge, experience and Machine Learning (ML) abilities. The first element is $l_{IT}(z)$ which is IT-specific labor- in other words so-called IT skills. The task-related experience and expertise of these specialists is the second element and is given by $\gamma_{IT}(z)$. The third element or ingredient in AI services is ML abilities, denoted A_{IT}. A_{IT} could, for instance, indicate the number or quality of software programs/algorithms available in the economy, reflecting for instance different ML techniques, see e.g. LeCun et al. (2015). As each software program has no rivalry in use the same program can be applied in each task. Therefore, the property of a software technology (ML ability) is contained in A_{IT}.

For an existing stock of AI technology the number and kind of tasks which are used and which fully substitute for labor (automation) will be endogenous. The relative factor prices and efficiency of these services will determine the extent of the use of automation technologies. Thus the degree of automation in this model is endogenous. In the next subsection this process is described in detail.

For now, it can be noted that if a task z with prize $p_h(z)$ is produced with pure labor $h(z) = A_L\gamma_L(z)l(z)$, and labor rewards are calculated according to marginal productivity, then $p_h(z)A_L\gamma_L(z) = w_L$. Symmetrically, the same task could be produced with an AI technology so that $p_h(z)A_{IT}\gamma_{IT}(z) = w_{IT}$. Given these two conditions, and given wages in the market, for any particular task the firm will choose choose the kind of production (automation or not) that results in the lowest unit labor costs. Thus, if the following condition holds, the task will be automated:

$$\frac{w_{IT}}{p_h(z)A_{IT}\gamma_{IT}(z)} < \frac{w_L}{p_h(z)A_L\gamma_L(z)}$$

This rule leads to condition (11) which identifies the switching point between automated

(AI) tasks and labor tasks. If tasks are ordered in such a way that $\frac{A_L\gamma_L(z)}{A_{IT}\gamma_{IT}(z)}$ is increasing in z and the tasks with lower numbers $z \in [N-1, N_{IT}]$ are the automated tasks, task N_{IT} is the switching point from an automation task to a labor task. N_{IT} is the highest number in this order for which

$$\frac{A_L\gamma_L(N_{IT})}{A_{IT}\gamma_{IT}(N_{IT})} < \frac{w_L}{w_{IT}} \tag{11}$$

holds. Apart from these automated (AI) tasks $[N-1, N_{IT}]$, all other tasks $(N_{IT}, N]$ are produced with standard labor. Thus, the costs and respectively the price $p_h(z)$ for any task z is

$$p_h(z) = \begin{cases} \frac{w_{IT}}{A_{IT}\gamma_{IT}(z)} & \text{if } z \in [N-1, N_{IT}] \\ \frac{w_L}{A_L\gamma_L(z)} & \text{if } z \in (N_{IT}, N] \end{cases} \tag{12}$$

2.2.2 Human service firm's optimization

Human services are produced by human-service firms who take the price for human services, the price for each task, and wages for various labor inputs, as given. It is assumed that these firms operate in competitive markets and that they will aim to maximize profits for a given price p_H subject to the production process in (9), such that

$$\pi_H = p_H H - p_h(z)h(z) = p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} - p_h(z)h(z)$$

From which the demand for task z can be derived to be:

$$h(z) = \frac{p_H^{\sigma} H}{p_h(z)^{\sigma}} \tag{13}$$

Combining (12) and (13) we can derive the demand for automation and labor tasks z as follows[5]

$$h(z) = \begin{cases} p_H^\sigma H \left(\frac{A_{IT}}{w_{IT}}\right)^\sigma \gamma_{IT}(z)^\sigma & \text{if } z \in [N-1, N_{IT}] \\ p_H^\sigma H \left(\frac{A_L}{w_L}\right)^\sigma \gamma_L(z)^\sigma & \text{if } z \in [N_{IT}, N] \end{cases} \tag{14}$$

Further, from (14) and (10) we can obtain the optimal demand for IT labor:

$$l_{IT}(z) = \begin{cases} \frac{p_H^\sigma H}{(w_{IT})^\sigma} (A_{IT})^{\sigma-1} \gamma_{IT}(z)^{\sigma-1} & \text{if } z \in [N-1, N_{IT}] \\ 0 & \text{if } z \in [N_{IT}, N] \end{cases} \tag{15}$$

and standard labor:

$$l_L(z) = \begin{cases} 0 & \text{if } z \in [N-1, N_{IT}] \\ \frac{p_H^\sigma H}{(w_L)^\sigma} (A_L)^{\sigma-1} \gamma_L(z)^{\sigma-1} & \text{if } z \in (N_{IT}, N] \end{cases} \tag{16}$$

Relative labor productivity can be determined from factor abundance and technology- and productivity-related parameters. Assuming that all types of labor are fully used in the various tasks, labor in all tasks add up to given total labor in each labor market segment

$$L_{IT} = \int_{N-1}^{N_{IT}} l_{IT}(z)dz, \text{ and} \tag{17}$$

$$L_L = \int_{N_{IT}}^{N} l_L(z)dz \tag{18}$$

By using (15), (16), (17) and 18) relative labor productivity is:

$$\frac{w_L}{w_{IT}} = \left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}} \left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \tag{19}$$

[5] For details see Appendix B.

2.2.3 Optimal number of automated tasks

Combining relative marginal productivity (19) with condition (11) and applying the *implicit function theorem* gives an expression for calculating the optimal number of automated tasks in the economy, which are endogenous:[6]

$$N_{IT} = N_{IT}\left(L_{IT}, L_L, A_{IT}, ...\right), \text{ with } \frac{dN_{IT}}{dL_{IT}} > 0, \quad \frac{dN_{IT}}{dL_L} < 0, \quad \frac{dN_{IT}}{dA_{IT}} > 0 \tag{20}$$

This expression indicates that the number of automated tasks crucially depends on the relative availability of the production factors as well as the availability of AI technologies. If IT labor is broadly available and hence its relative wage low, more tasks could be automated.

Similarly, if IT knowledge and AI algorithms are readily available, relative wages $\frac{w_L}{w_{IT}}$ increase and make standard labor tasks relatively more expensive. This results in a higher share of automated tasks. The clear implication is that if an economy is advanced in terms of IT technologies and IT labor, this economy will be more automated.

2.2.4 Optimal human service supply

From the demands for the various tasks, total human service production can be derived. Aggregating automated tasks and labor, equation (9) leads to

$$H = \left(\int_{N-1}^{N_{IT}} h(z)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}$$

[6] For details see the Appendix B.

Using (14), (84) and 85), respectively, and re-arranging gives the expression for total production of human services as:[7]

$$H = \left(\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_{IT} L_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\int_{N_{IT}}^{N} \gamma_{L}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

In order to simplify this expression Acemoglu and Restrepo (2018a,b) and Acemoglu and Restrepo (2019b) propose two definitions that allow for a more compact expression. With the definitions

$$\Gamma(N_{IT}, N) = \frac{\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz} \tag{21}$$

and

$$\Pi(N_{IT}, N) = \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma-1}} \tag{22}$$

one may substitute $B_1(N_{IT}) = \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz = (1 - \Gamma(N_{IT}, N))\Pi(N_{IT}, N)^{\sigma-1}$ and $B_2(N_{IT}) = \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$ and thus rewrite the aggregate optimal human service production as

$$H = \Pi(N_{IT}, N) \left[(1 - \Gamma(N_{IT}, N))^{\frac{1}{\sigma}} (A_{IT} L_{IT})^{\frac{\sigma-1}{\sigma}} + \Gamma(N_{IT}, N)^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{23}$$

This expression is similar to the familiar Constant Elasticity of Supply (CES) production function.

2.2.5 Earning shares of laborers

From equation (23) the earning share of each group of L_L and L_{IT} can be deduced. After rearranging these, the earning share of standard labor from revenues earned by human services can be written as:[8]

[7] For details see Appendix B.
[8] For details see Appendix B.

$$
\begin{aligned}
\phi_L &= \frac{w_L L_L}{p_H H} = \frac{1}{1+\left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{\frac{1}{\sigma}}\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}} \quad \text{(non-IT labor)}, \qquad (24)\\
\phi_{IT} &= \frac{w_{IT} L_{IT}}{p_H H} = 1-\phi_L \quad \text{(IT labor)}
\end{aligned}
$$

2.3 Intermediate goods-producing firms

In our model we have final-good producing firms (whose final goods production under market frictions was set out above in section 2.1), as well as human-service firms who produce human services, described in the previous section (2.2). The third group of firms consists of firms producing intermediate goods that are used by final goods-producing firms. In this subsection we describe these firms in greater detail.

2.3.1 Market entry of intermediate goods-producing firms

The intermediate goods-supplying firms in our model are monopolists because they each sell an unique product which is the outcome of entrepreneurial (product) innovation. The costs for the typical firm (denominated in units of final output) to produce one unit of x is c_x, and the profits this result in is $\pi_x = (p_x - c_x)\,x$.

Using the demand function (8) and plugging in $p_x = \alpha H_Q^{1-\alpha} x^{-(1-\alpha)}$ results in:

$$
\pi_x(t) = \alpha H_Q^{1-\alpha} x(t)^{-(1-\alpha)} x(t) - c_x x(t) \qquad (25)
$$

From the first-order condition[9] and using (8) and (27), the optimal price p_x and optimal

[9] The first-order condition is $\frac{\partial \pi_x}{\partial x} = \alpha^2(1-\theta_i)H^{1-\alpha}x^{\alpha-1} - c_x = 0$, thus $c_x = \alpha^2(1-\theta_i)H^{1-\alpha}x^{\alpha-1} \Leftrightarrow x^{1-\alpha} = (c_x)^{-1}\alpha^2(1-\theta_i)L^{1-\alpha}$.

production of intermediate goods $x\left(t\right)$ are, respectively:

$$p_x = \frac{c_x}{\alpha} \tag{26}$$

and

$$x\left(t\right) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q \tag{27}$$

Given (27) and (26), maximum profits $\pi_x\left(t\right)$ are:

$$\pi_x\left(t\right) = \left(\frac{1}{\alpha} - 1\right)\left(c_x\right)^{\frac{-\alpha}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} H_Q \tag{28}$$

The present value of this future profit flow, discounted at the steady-state interest rate r, is:

$$V_x(t) = \frac{1}{r}\pi_x\left(t\right) = \int_t^{\infty} \pi_x\left(t\right) e^{-r(v,t)(v-t)} dv \tag{29}$$

Here, $\frac{1}{r}\pi_x$ is the present value of profits per innovation and $\frac{1}{r}\pi_x\dot{N}$ are the total profits of the intermediate goods producing firm (which is essentially a new firm) of introducing $\dot{N}(t)$ new goods. In addition to the cost of innovation, the new firm also has to cover the costs of market entry (e.g., commercialization costs) for the new intermediate good, which is ν. Thus, the total entry cost of the start-up with innovation rate $\dot{N}$ and thus total investment is

$$\dot{N}\nu = I \tag{30}$$

With competitive market entry, the net rents of a new firm turn to zero and the net present value of the new firm just about covers its total start-up costs:

$$\frac{1}{r}\pi_x(t)\dot{N}(t) - I(t) = 0 \tag{31}$$

With $\dot{N}\nu = I$ the steady-state interest rate is:

$$r = \frac{\pi_x(t)}{\nu} \tag{32}$$

2.3.2 Supply of innovative intermediate products

Innovation in the intermediate goods market is exogenously given as $\dot{A}(t) = \frac{dA(t)}{dt}$, which is the number of innovative intermediate products invented at t. These innovative intermediate products are not automatically successful in the market. The success or failure to find a buyer can be modelled as an *aggregate matching* process.[10]

In such a matching process, the number of new intermediate products successfully entering the market $\dot{N}$ is a function of two elements: (i) the given number of new, innovative intermediate products $\dot{A}(t)$ potentially ready for market entry, and (ii) the number of opportunities for market entry that entrepreneurs (start-ups) discover. These opportunities are determined by the capacity of the market. Absorption capacity for intermediate goods is a function of total effective demand for intermediate goods in the economy $X^{eD}(t)$.

Through an aggregate matching function, these two elements can be combined and the resulting process of market entry can be described as $\dot{N} = f(\dot{A}, X^{eD})$. For simplicity, it is assumed here that the matching technology is subject to constant economies of scale, so that the number of new products in the market will be given by

[10]For a micro-foundation of this process see Gries and Naudé (2011).

$$\dot{N}(t) = \left(X^{eD}(t)\right)^{\varphi} (\dot{A}(t))^{1-\varphi} \tag{33}$$

where φ is the contribution of market opportunities. Although the assumption of a macro-matching process is basic, it represents the main idea behind the mechanism. Given (33), the growth of new products in the economy is a *semi*-endogenous process because the number of new products $\dot{A}$ is fixed but the number of new technologies implemented to establish intermediate products $\dot{N}$ is endogenous.

2.4 Aggregate production and income distribution

Having specified final goods and intermediate goods production, and in having showed how the task approach can be used to account for human service production in the preceding sections, this sub-section is concerned with the aggregate budget constraint and the distribution of income to the various agents in the economy, starting with labor income (2.4.1).

2.4.1 Labor income

As we discussed in the preceding sections, human services H are allocated to two activities, namely production H_Q and sales promotion H_ϕ, $H = H_Q + H_\phi^*$. For the representative firm $H_{\phi i}^*$ has already been determined by condition (5) and (6). Thus, the allocation of human services to production must be

$$H_Q = H - H_\phi \tag{34}$$

From (7) we know that human service in production is paid according to its marginal productivity with the price $p'_H(t)$. However, not only do firms have to pay human services used in physical production H_Q, they also need to pay human services used in sales promotion H_ϕ. As factor rewards are paid in physical output goods at an amount $(1-\alpha)Q$, all human services needs to be paid out of this amount. Also, with a homogeneous H in a perfectly integrated human service market, only one price is paid to to H, irrespective of whether used in production or sales promotion.

Finally, because total payment for H cannot exceed the contribution of H to effective production (7), we obtain an average income that is paid to all human services. Thus, total or aggregate human service income is $p_H H = p'_H H_Q = (1-\alpha)Q$[11] and the price for H is

$$p_H(t) = (1-\alpha)\frac{Q(t)}{H}. \tag{35}$$

2.4.2 Wealth holders' income

$N(t)\pi_x(t)$ denotes total debt issued in the economy. All new products, results of innovation (R&D), are financed by issuing new debt, $\dot{N}(t)\nu = \dot{F}(t)$. As wealth holders, profits accrue to the owners of this debt - the financiers:

$$N(t)\pi_x(t) = r(t)F(t) \tag{36}$$

2.4.3 Production and income constraints

Effective output in the economy has to be divided amongst intermediate goods x, standard labor L_L, and the IT technology service provider L_{IT}. The budget constraint for effective output is therefore

$$Q(t) = N(t)H_Q^{1-\alpha}x^{\alpha}(t) = N(t)p_x(t)x(t) + w_L(t)L_L + w_{IT}(t)L_{IT} \tag{37}$$

[11] $p'_H H_Q = p_H(H_Q + H_\Phi) \Leftrightarrow p_H = p'_H \frac{H_Q}{(H_Q+H_\Phi)} = (1-\alpha)\frac{Q}{H_Q}\frac{H_Q}{(H_Q+H_\Phi)} = (1-\alpha)\frac{Q}{H}$. Note that Q depends on H_Q.

Note that effective output is not the same as GDP or aggregate income. As x is produced by using c_x units of final goods, net final output and thus *income* is

$$Y(t) = Q(t) - N(t)x(t)c_x \tag{38}$$

Further, (37) and (38) imply that $Q - Nxc_x = Np_x x - Nxc_x + w_L L_L + w_{IT} L_{IT}$. With the definition of profits in the intermediate goods sector (25), the income constraint then becomes:

$$Y(t) = N(t)\pi_x(t) + w_L(t)L_L + w_{IT}(t)L_{IT} \tag{39}$$

According to (39) total or aggregate income in the economy consists of profits, labour, and technology income. Given equation (32) this means that $Y = rN\nu_x + w_L(t)L_L + w_{IT}(t)L_{IT}$. Value added generated by innovative intermediate firms therefore turns into the income of financial asset owners $r\,(t)\,F\,(t)$. The growth process is thus essentially a process of financial wealth accumulation through the financing of new products and (intermediate-good producing) new ventures. It may be labelled a "Silicon Valley" model of growth.

Finally, using (36) results in the familiar income decomposition of GDP:

$$Y\,(t) = r\,(t)\,F\,(t) + w_L(t)L_L + w_{IT}(t)L_{IT} \tag{40}$$

In addition to income of financial wealth owners, value added generated by the human service input is distributed to labour ($w_L(t)L_L$) and the providers of the AI technologies and services ($w_{IT}(t)L_{IT}$).

2.4.4 Income distribution

To allow us to eventually trace the distributional consequences of progress in artificial intelligence (AI), the income shares of the three input and resource providing agents in the model need to be derived. These are the income of standard labor ($w_L(t)L_L$), the AI service providers ($w_{IT}(t)L_{IT}$), and the financial investors ($r(t)F(t)$).

Wages and income share of labor: Using the expression for factor demand (35) and (24), wages can be related to total income as follows:

$$w_L = \phi_L \frac{p_H H}{L_L} = \frac{\phi_L}{1+\alpha} \frac{Y(t)}{L_L} \tag{41}$$

Because ϕ_L is constant, w_L is the standard wage rate in the economy. The income share of standard labour can now be derived by using (41), (27) and (38) as:[12]

$$\frac{w_L(t)L_L}{Y(t)} = \frac{\phi_L}{(1+\alpha)} < 1 \tag{42}$$

Wages and income share of the AI provider: The factor reward, or wage rate, of the economic agent that provides the AI at amount A_{IT} can be derived in a symmetrical manner as in (41) and can thus be specified as:

$$w_{IT} = (1-\phi_L) \frac{p_H H}{L_{IT}} = \frac{1-\phi_L}{1+\alpha} \frac{Y(t)}{L_{IT}} \tag{43}$$

The income share of providers of the AI service is accordingly:

$$\frac{w_{IT}(t) L_{IT}}{Y(t)} = \frac{1-\phi_L}{1+\alpha} < 1 \tag{44}$$

[12]Note that $\frac{1-\alpha}{1-\alpha^2} = \frac{1}{(1+\alpha)}$. See also Appendix C.

Income share of financial investors: The income share of financial investors can be calculated using (38), (36), and (25) as [13]

$$\frac{N(t)\,\pi_x(t)}{Y(t)} = \frac{\alpha}{1+\alpha} \tag{45}$$

2.5 Aggregate expenditure and income

To understand and analyze the role of aggregate demand it is necessary to specify the consumption and savings behaviour of the agents in the economy.

In standard endogenous growth models, aggregate demand is typically modelled assuming *representative* intertemporal choices based on a *representative* household's Euler equation.[14] This, however, is not adequate when asymmetries in factor rewards and potential changes in income distribution are key features of interest - as is the case when considering automation technology. The representative household assumption in standard endogenous growth models assumes away differences in intertemporal decisions of rich and poor households and their respective effects on aggregate consumption and savings. In Appendix D examples are provided for specific intertemporal choices at individual or group level. Moreover, if group preferences are heterogeneous, they may lead to heterogeneous consumption and savings behaviour which needs to be taken into consideration given that it specifies that effective aggregate supply and demand for intermediate inputs depends on aggregate demand.

The novel model proposed here does not assume away the idea of rational intertemporal choices, as is usually the case in endogenous growth models. However, what it does reject is the idea of a simple aggregation rule like a representative household (Gries, 2020b). Instead, for present purposes the Keynesian tradition is followed by assuming that some households

[13] Details of the calculation are contained in Appendix C.

[14] $\frac{\dot{C}}{C} = \frac{r_D - \rho}{\eta_U}$ with ρ denoting the representative agent's time preference rate and η_U the intertemporal elasticity of substitution.

only earn wage income $w_L L_L$ and another group of households earn only financial income from assets rF. A third group, providers of AI-services, is also regarded as its own group. Each group has its own consumption preferences and patterns. Labor income accrues to poorer households while financial wealth holders and AI service providers accrue income for richer households.

2.5.1 Consumption and investment expenditure

From (42) we know the share of labor income. We define group-specific intertemporal choice and assume plausible group-specific parameters for the choice problem, and assume that total wage income is fully consumed, and that labor income is the only source of consumption expenditure. The latter is a traditional assumption in Keynesian growth models (Gries, 2020b). In Appendix D we show that motivating this assumption by suggesting group-specific optimal intertemporal choices is not difficult. The important assumption is that groups are different and have different expenditure behavior.

According to (42) the share of labor income is $\frac{w_L L_L}{Y} = \frac{\phi_L}{(1+\alpha)}$. As labor belongs to poorer households, it is assumed here that total wage income will be fully consumed and that total wage income is the only source of consumption expenditure.[15] Further, in an economy with non-perfect matching, consumers also devote income to search and matching activities whenever their desired consumption cannot find a suitable output. Searching for appropriate consumption goods leads to the experience that using fraction θ_j of their income in the search

[15]In Appendix D we show that once we depart from the representative household approach, motivating this assumption by group-specific optimal intertemporal choices is not difficult.

and matching procedure would reduce the mismatch.[16] Therefore aggregate consumption is:

$$C(t) = w_L(t) L_L (1-\theta) + \varepsilon = c(1-\theta) Y(t) + \varepsilon \tag{46}$$

$$\text{with} \quad c = \frac{\phi_L}{(1+\alpha)} \tag{47}$$

Note here that c is the economy's marginal (and average) rate of consumption. ε denotes a randomness in consumption demand with an expected value $E[\varepsilon] = 0$.

As far as investment expenditure is concerned, in our model the innovation by intermediate-goods producing start-up ventures requires investment[17]. It is assumed that such investment ν is identical for each innovation. Thus total start-up investments $I(t)$ are described by

$$I(t) = \nu \dot{N}(t) \tag{48}$$

2.5.2 The Keynesian income-expenditure equilibrium

Income Y can be used for consumption C and investment I. Thus demand for GDP is $Y^D \equiv C + I$. While the consumption rate is determined by (46) and a constant fraction of total effective income, investments are driven only by the market entry of new goods (i.e., innovation), $\dot{N}$. With the consumption rate (47) being a constant, the Keynesian income-expenditure mechanism can be applied to determine effective total demand, Y^D. Therefore, in income-expenditure equilibrium, aggregate effective demand equals effective income

$$Y(t) \overset{!}{=} Y^D(t) \equiv C(t) + I(t), \tag{49}$$

[16] In section 3.2 when we introduce the aggregate *match-improvement function* (56) we will see how θ affects the matching process. This simple way of modeling the consumers' search activity implies that subtracting search costs theta from income is a kind of iceberg cost of this search.

[17] Note that the term investment stands for start-up expenditure on final output goods. It is not a capital formation that accumulates to a stock of real capital for production purposes.

and we obtain the Keynesian income-expenditure multiplier for the effective expected demand in aggregate goods market

$$Y^D(t) = \frac{I(t) + \varepsilon(t)}{1 - c(1-\theta)} = \frac{\nu \dot{N}(t) + \varepsilon(t)}{1 - c(1-\theta)} \tag{50}$$

2.5.3 Expected aggregate demand for total production

To determine the total or aggregate demand for final output Q, we begin with the demand for GDP, $Y^D(t) \equiv C(t) + I(t)$. We also need to add the demand for input goods taken from final goods sector $N(t)x(t)c_x$. The Keynesian income-expenditure mechanism tells us that effective aggregate demand for GDP is $\frac{\nu \dot{N} + \varepsilon}{1-c(1-\theta)}$, adding $N(t)x(t)c_x$ gives the effective demand for total output Q, namely

$$Q^D = \frac{\nu \dot{N}(t) + E[\varepsilon(t)]}{1 - c(1-\theta)} + N(t)x(t)c_x$$

Demand is hence an endogenous value in which investment expenditures are independent from households' savings decisions. Further, to determine the expected excess demand ratio under current demand conditions, we need to divide by $Q(t)$. As a result, the aggregate effective demand ratio $\lambda(t)$ describes the ratio of effective aggregate demand to current output

$$\lambda(t) = \frac{Q^D(t)}{Q(t)}$$

and in expected values we obtain the *ratio of expected aggregate demand* [18]

$$E[\lambda(t)] = \frac{\nu}{1 - c(1-\theta)} \frac{1}{H_Q^* \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} g_N + \alpha^2 \tag{51}$$

[18] $\frac{E[Q^D]}{Q} = \frac{\nu}{1-c(1-\theta)} \frac{\dot{N}(t)}{Q(t)} + \alpha^2$ and using (1) and (27) we obtain (51).

3 Solving the model

In this section, we depart from the perspective of individual firms and consumers and assume the perspective of an omniscient observer of the economy.

3.1 Solving for technology growth

We start to solve the model by determining the semi-endogenous growth rate of new products that successfully enter and remain in the market $g_N(t) = \frac{\dot{N}(t)}{N(t)}$. From Equation (33) we know that the growth rate of implemented technologies depends on effective demand for intermediate goods and thus depends on labor in effective production H_Q, and is

$$g_N(t) = \frac{\dot{N}(t)}{N(t)} = \left(\left(\frac{\alpha^2}{c_x} \right)^{\frac{1}{1-\alpha}} H_Q \right)^{\varphi} (g_A)^{1-\varphi} \tag{52}$$

This process is semi-endogenous, as the exogenous g_A is an essential driver of g_N. However, the extent to which the exogenous innovative process g_A becomes usable and implemented in the economy is endogenous.

3.2 From perceived individual frictions to aggregate market mismatch

In section 2.1 we introduced the notion of a firm facing market friction in selling its potential output.[19] From the perspective of an individual firm i, we have discussed firm i's perception of market mismatch δ_i which they relate to their individual market conditions and their counter-activities. They use human services $H_{\phi i}$ for placement and reduce their individual

[19] This section is closely related to the modelling in Gries (2020a).

sales problems accordingly. Furthermore, in the preceding sections we explained that it is not only firms that are affected by a market mismatch. In their search for the desired consumption goods, consumers also face a mismatch and hence devote a fraction θ_j of their income on this search.

These micro-level (idiosyncratic) problems in the market are not the only reason for firms' sales and customers' purchase problems. These problems are in fact, also due to aggregate market conditions, even if individual decision-makers are not aware of this fact.

What are the reasons for firms' sales problems? From the perspective of firms, effective sales are determined by stochastic market mismatch δ_i, $[\phi_i(t) = 1 - \delta_i\,(t)$ see 2]. Thus, to answer this question we need to find out more about the random variable $\delta_i\,(t)$. Furthermore, what is behind the firm's perceived market frictions $\delta_i'\,(t)$?

The mismatch is clearly determined by two components, (i) aggregate market conditions and (ii) an idiosyncratic component for each individual firm.

The first component, aggregate market conditions, reflects a shortage of aggregate demand $\delta^D\,(t)$ - which is the difference between total supply and effective aggregate demand $Q^D(t)$

$$\delta^D\,(t) = \frac{Q(t) - Q^D(t)}{Q(t)} = 1 - \lambda(t) \tag{53}$$

A second and additional component of the mismatch is the idiosyncratic component for each firm. Sales problems are firm-specific obstacles and are described by the random variable ε_{Fi}, with $1 > E[\varepsilon_{Fi}] > 0$. For given aggregate market conditions $\delta^D\,(t)$, ε_{Fi} is the element of the mismatch that is due to individual firm conditions. Therefore, the friction perceived by each firm i combines the aggregate market and idiosyncratic component and can be described as

$$\delta_i'\,(t) = \delta^D\,(t)\,\varepsilon_{Fi} \tag{54}$$

However, individual firms or consumer do not have this insight into the breakdown of the friction. An individual firm only perceives an expected sales ratio $E\left[\phi_i\left(t\right)\right] = 1 - E\left[\delta_i\left(t\right)\right]$, interpreting it as being caused by a friction that can be addressed by allocating more labor towards the matching process [$\frac{\partial E[\phi_i(t)]}{\partial E\left[\delta_i'\right]} < 0, \quad \frac{\partial E[\phi_i(t)]}{\partial H_{\phi i}(t)} > 0$ see (3) section 2.1].

We have to aggregate to connect these individual activities with total and current market conditions to determine aggregate market equilibrium. Assuming that ε_{Fi} are i.d for $i \in \mathcal{I}$, we can aggregate ($\varepsilon_{Fi} = \varepsilon_F$,) and obtain as general or representative perceived friction $\delta'\left(t\right)$; and in expectations

$$E\left[\delta'\left(t\right)\right] = \left(1 - E\left[\lambda\right]\right) E\left[\varepsilon_F\right] - cov\left(\lambda, \varepsilon_F\right), \quad \text{with} \quad cov\left(\lambda, \varepsilon_F\right) < 0 \tag{55}$$

This shows the full mechanism that leads to mismatches. Further, we assume that $cov\left(\lambda, \varepsilon_F\right)$ sufficiently lalrge in absolute terms such that $E\left[\delta'\left(t\right)\right]$ is always positive. However, we have not specified how counter-measures by firms and customers affect the mismatch. To do this, we define the aggregate match-improvement function $m(t)$ for the aggregate market. We assume that matching of the two market sides is determined by the firms' allocation of human services to combat mismatch $H_{\phi i}\left(t\right)$ and of the fraction $\theta\left(t\right)$ of consumers' income spent to find the desired consumption good

$$m = L_\phi\left(t\right)\left(1 - \theta\left(t\right)\right)^{-1}, \quad \text{with} \ \frac{dm}{dH_\phi} > 0, \quad \frac{dm}{d\theta} > 0 \tag{56}$$

Thus, the rate of expected effective aggregate mismatch -after implementing counter-measures - is

$$E\left[\delta\left(t\right)\right] = E\left[\delta'\left(t\right)\right] - m \tag{57}$$

When the mismatch is completely eliminated, such that the aggregate expected mismatch becomes zero, we obtain a perfect matching

$$E\left[\delta\left(t\right)\right]=0 \tag{58}$$

Thus, (i) equation (58) implies that firms are in sales equilibrium as the expected effective sales ratio turns to one,

$$1=E\left[\phi\left(t\right)\right] \tag{59}$$

While (59) simply defines the condition for a production and sales process that is free of any mismatch, we simultaneously need to determine aggregate market equilibrium. For the aggregate market we know that effective sales must be equal to the effective aggregate demand, and in aggregate market equilibrium both must equal aggregate supply $E\left[\Phi\right]=E\left[Q^D\right]=Q$. Thus, as the second equilibrium condition we obtain

$$1=\frac{E\left[Q^D\right]}{Q}=E\left[\lambda(t)\right] \tag{60}$$

3.3 The aggregate model in two equations

Using (52) reduces the system to the following two simultaneous equations, namely (59a) and (60a).

3.3.1 Firms' sales equilibrium

From (5) in section 2.1 we know that a firm allocates human services in the market placement process until all output is sold. On aggregate (57) and (59) tell us that producers and customers allocate resources to improving aggregate matching until all production is sold. Using the constraint for human service allocation (34) we can now state firms' sales equilibrium for the representative producer as[20]

$$H_Q = cov\left(\lambda, \varepsilon_F\right)\left(1-\theta\right) + H \tag{59a}$$

3.3.2 Aggregate market equilibrium

In section 2.5 we specified aggregate demand and the aggregate effective demand ratio (see 51). Aggregate goods market equilibrium requires that demand equals production and supply, such that the effective demand ratio turns to one and plugging in (60) and (52) gives[21]

$$H_Q = \left(\frac{\nu}{\left(1-\alpha^2\right)\left(1-c\left(1-\theta\right)\right)}\right)^{\frac{1}{(1-\varphi)}} \left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{(1-\alpha)(1-\varphi)}} g_A \tag{60a}$$

Two equations (59a) and (60a) hence remain to solve for the two endogenous variables, namely human services used in production H_Q and consumers' spending on search and matching θ.

3.4 Current market equilibrium

We can now solve for equilibrium. Combining (59a) with (60a) we are left with only equation (61) and one variable, H_Q

$$0 = F = H_Q^{(1-\varphi)} - \frac{\nu}{\left(1-\alpha^2\right)\left(1-c\frac{H-H_Q}{-cov(\lambda,\varepsilon_F)}\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(g_A\right)^{1-\varphi} \tag{61}$$

As we cannot explicitly solve for H_Q, we apply the *implicit function theorem* to determine the equilibrium $\tilde{H}_Q$, and other interesting variables.

[20] For details see Appendix E.
[21] For details see Appendix E.

Proposition 1 ***Current market equilibrium:*** *Equation (61) implicitly defines a function for*

(i) the equilibrium value of $\tilde{H}_Q$

$$\tilde{H}_Q = H_Q\left(\nu, g_A, c_x, A_{IT}, ..., cov\left(\lambda, \varepsilon_F\right)\right), \text{ with } \frac{d\tilde{H}_Q}{dg_A} > 0, \qquad \frac{d\tilde{H}_Q}{d\nu} > 0 \tag{62}$$

Further, (62) leads to

(ii) the rate of consumers' spending on improving the matching process

$$\tilde{\theta} = 1 - \frac{H - \tilde{H}_Q}{-cov\left(\lambda, \varepsilon_F\right)}, \quad \text{with} \quad cov\left(\lambda, \varepsilon_F\right) < 0 \tag{63}$$

(iii) total production of the final good

$$\tilde{Q}(t) = N\left(t\right)\tilde{H}_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}, \tag{64}$$

(iv) total income and hence the level of the growth path

$$\tilde{Y}(t) = N\left(t\right)\left(1-\alpha^2\right)\tilde{H}_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}, \tag{65}$$

(v) the growth rate of income (GDP) gives

$$\tilde{g}_Y = \frac{\dot{Y}\left(t\right)}{Y\left(t\right)} = g_N = \frac{\dot{N}(t)}{N(t)} = \left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi}{1-\alpha}}\left(\tilde{H}_Q\right)^{\varphi}\left(g_A\right)^{1-\varphi}, \tag{66}$$

and (vi) the real rate of return on financial investment

$$\tilde{r} = g_N - \frac{\left(1-\phi_L\right)\left(1-\alpha\right)\alpha^{\frac{2\alpha}{1-\alpha}}c_x^{-\frac{\alpha}{1-\alpha}}H_Q}{\nu} \tag{67}$$

For a proof, see Appendix F.

With $\tilde{H}_Q$ and $\tilde{H}_\phi$ we have determined a current market equilibrium at a level below potential output $\tilde{H}_Q < H$. Further, as $\tilde{H}_Q$ depends on demand side parameters, e.g. ν, the level of the income path and the growth rate is restricted by the demand side. It is also interesting to note that in this kind of economy, the return on investment is equal to the growth rate.

While (67) is a result that is also found in other mainstream models, causality is different. In this model g_N is the driver of r. As more products enter the market, profits improve and the return on investments increases. In endogenous growth theory, r is the result of an intertemporal choice and drives both the growth rate and the savings rate.

3.5 Stationarity of Equilibrium

Although market equilibrium for each period is described in section 3.4, two important questions remain. First, how can the equilibrium output steadily remain below potential output and represent a long-term stationary equilibrium? Second, how can aggregate demand become central and determine both the stationary level and the speed of the growth path? The next two subsections provide the answers.

These questions are worth asking because mainstream dynamic macroeconomics is based on the idea that the path of potential growth - often regarded as the outcome of some kind (variety) of neoclassical or endogenous growth model - is the only relevant process for economic growth. After a temporary deviation from this path, the economy returns to it and continues to grow as described in the fundamental growth model. There is no permanent deviation. In contrast, the equilibrium we derived here can indeed become a permanent, stationary process. In other words, such a demand-restricted growth path (path level and growth rate) is a stationary path. The economy will not necessarily return to the path of potential growth.

3.5.1 The stationary no-expectation-error equilibrium

In this approach we suggest a different concept for a stationary equilibrium[22] that is directly related to stochastic modeling. We describe stationary behavior from the perspective of individual decision-makers.

We assume that a systematic difference of expectations and planning with the average outcome of a stochastic variable is perceived as an inconsistency of one's own behavior and reality that leads to an adjustment in behavior towards an experience of less inconsistency. For instance, if in a stochastic environment an individual plans and organizes a specific outcome - according to their subjective expectations - and their plans and outcome do not coincide with observed expected values, we refer to this difference as an ***expectation error***. As a consequence the individual learns from this error and changes their behavior by adjusting their plans. Individual behavior becomes stationary if the planned and realized outcome is indeed the observed expected outcome. This condition defines a behavioral equilibrium such that it implies no (need) for a change in behavior. Thus, we refer to this condition as the ***no-expectation-error equilibrium*** (n-e-ee). It is an equilibrium in terms of a stationary behavior.

In this approach, the general concept of a no-expectation-error equilibrium can be illustrated by looking at the matching procedure. The mismatch $E[\delta]$ defines the gap between planned production $Q_i(t)$ and the mean of effective sales $E[\Phi_i(t)] = (1 - E[\delta(t)])\, Q_i(t)$. Thus, as long as firms and customers do not counter the mismatch by devoting sufficient resources to the matching they cannot expect the mismatch to disappear, and $E[\delta] = E[\delta'] - m(L_\phi, \theta) > 0$. Thus, individuals face an expectation error as their actions do not coincide with the observed expected values. In other words, there is an error in their planning as their subjective expectations are false (expectation error). Thus, they continue to adjust their plans until

[22] This equilibrium concept draws on Gries (2020b).

they correctly expect and plan their counter-activities, such that the expected mismatch is on average fully eliminated $0 = E[\delta] = E[\delta'] - m(L_\phi, \theta)$. As a result, there is no expectation error with respect to the final goods matching mechanism. Firms are in sales equilibrium ($E[\phi] = 1$). There is also no expectation error with respect to consumers and aggregate markets. In aggregate market equilibrium consumers allocate income share θ to the search process and find a matching equilibrium for all their income planned for consumption, such that we also obtain equilibrium in the aggregate goods market $E[\lambda(t)] = 1$.

Definition 1: No-expectation-error equilibrium. *Firms and customers are in "no-expectation-error equilibrium" (n-e-ee) if (i) the expected mismatch is correctly predicted, such that respective planned counter-activities fully eliminate the expected mismatch*

$$E[\delta'] = m(L_\phi, \theta) \tag{68}$$

and (ii) furthermore, firms and customers exhibit stationary behavior (no change in behavior is necessary) as they expect what they plan and realize, such that firms remain in sales equilibrium and the aggregate market continues to remain in market equilibrium,

$$\begin{aligned} E[\phi] &= 1, \text{ see (59)} \\ E[\lambda] &= 1, \text{ see (60)} \end{aligned}$$

Using Definition 1 above we see that the equilibrium which is determined in proposition 1 is indeed a stationary equilibrium. Thus, we can state the following proposition.

Proposition 2 ***Steady state equilibrium:*** *The market equilibrium derived in proposition 1 is a no-expectation-error equilibrium and thus a stationary equilibrium.*[23]

[23]Proof: As (62) and (63) in proposition 1 satisfies condition (58), (59) and (60), respectively, conditions of Definition 1 are satisfied. Thus, firms and customers are in a no-expectation-error equilibrium and exhibit stationary behavior.

This outcome, a demand-restricted stationary growth path below the level of potential growth, is the most significant difference in our model from mainstream endogenous growth models, and a fundamental contribution of this paper.

Such as demand-restricted stationary growth path could not occur under the perfect market conditions of typical endogenous growth models. There are two reasons why and how can this happen here. First, firms observe a market mismatch which provides incentives for firms and customers to act. Both can respond to this perceived mismatch by allocating resources to reduce perceived frictions and improve the match between demand and supply. In response, labor potentially available for production is allocated to improve the matching process and expenditure that is potentially usable for consumption demand is spent on the search. This resource reallocation leaves the economy below the potential production level.

Having proposed an endogenous growth model with aggregate demand constraints and that incorporates the task approach, the next section uses this model to analyze the dynamic, long-run impacts of artificial intelligence.

4 The Dynamic, Long-Run Impacts of AI

The previous section of this paper presented an appropriate endogenous growth model to identify labor market and growth consequences of technological progress in AI. In this section the model is used to analyze the long-run impacts of AI. For simplicity we assume a once-and-for-all increase in the availability of AI technologies reflected by A_{IT}.

4.1 Impact on the number of automated tasks

The first effect we consider is the impact of an increasing AI availability ($\frac{dA_{IT}}{A_{IT}} > 0$) on the number of automated tasks. Taking the derivative of (20) we obtain[24]

$$\frac{dN_{IT}}{N_{IT}} = \eta_{N_{IT},A_{IT}} \frac{dA_{IT}}{A_{IT}} > 0 \tag{69}$$

with

$$\eta_{N_{IT},A_{IT}} = \frac{1}{\sigma \frac{\partial \gamma(N_{IT})}{\partial N_{IT}} \left(\frac{A_L L_L}{A_{IT} L_{IT}} \frac{B_1(N_{IT})}{B_2(N_{IT})} \right)^{\frac{1}{\sigma}} + \frac{\gamma_L (N_{IT})^{\sigma-1}}{B_2(N_{IT})} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})}} \frac{1}{N_{IT}} > 0. \tag{70}$$

Thus, with more AI, more tasks will be automated.

4.2 Impact on human service production

Progress in AI ($\frac{dA_{IT}}{A_{IT}} > 0$) will increase the supply of human services inputs H

$$\frac{dH}{H} = \eta_{H,A_{IT}} \frac{dA_{IT}}{A_{IT}} > 0, \quad \text{with} \quad \eta_{H,A_{IT}} > 0, \quad \text{for} \quad \sigma > 1 \tag{71}$$

given that[25]

$$1 > \eta_{H,A_{IT}} = \left[1 + \left(\frac{B_2\left(N_{IT}\right)}{B_1\left(N_{IT}\right)} \right)^{\frac{1}{\sigma}} \left(\frac{A_L L_L}{A_{IT} L_{IT}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{-1} > 0 \tag{72}$$

Thus, the supply of human services inputs will increase and at a rate slower than the rate of AI change $\frac{dA_{IT}}{A_{IT}}$. This is a similar result as in a standard constant elasticity of substitution (CES) approach, as in Gries and Naudé (2018).

[24]See appendix B1.
[25]For calculations see Appendix H.

4.3 Impact on inequality

To analyze the impact of AI ($\frac{dA_{IT}}{A_{IT}}$) on inequality, the changes it brings about in the income share of labor, the income share of the technology providers, and the income share of financial wealth holders will be determined.

Income share of labor: The income share of labor was described in equation (42). The derivative of (24) shows how the income share of labor income changes as a result of new AI technologies:[26]

$$\frac{d\left(w_L L_L/Y\right)}{w_L L_L/Y} = \frac{d\phi_L}{\phi_L} = \eta_{\phi_L,A_{IT}} \frac{dA_{IT}}{A_{IT}} < 0, \text{ for } 1 < \sigma \tag{73}$$

$$\text{with } \eta_{\phi_L,A_{IT}} = \frac{1}{\sigma} \frac{-(\sigma-1) - \left(\frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})} + \frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})}\right) N_{IT}\eta_{N_{IT},A_{IT}}}{1 + \left(\frac{A_L L_L}{A_{IT} L_{IT}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}}} < 0 \tag{74}$$

For $1 < \sigma$ (high elasticity of substitution) the income share of labor will clearly decline since $-\left(\frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})} + \frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})}\right) N_{IT}\eta_{N_{IT},A_{IT}} < 0$. However, if $1 > \sigma$ the income share of labor will not necessarily increase, in contrast to what would be the case in a standard CES approach. This shows that the integration of the task-based approach in the model means that automation will likely decrease the share of labor income, even if σ is low. This is because $-\left(\frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})} + \frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})}\right) N_{IT}\eta_{N_{IT},A_{IT}}$ is always negative and for not to large levels of σ the negative effect ay still dominate. This effect is the same as that identified by Acemoglu and Restrepo (2019b, p.9).

It can also be noted that A_{IT} progress will not only depress the income share that labor receives from providing human service inputs (for $1 < \sigma$), but that the share of labor income in the *total economy* will also decline.

[26]For calculations see Appendix H.

Income share of technology providers: Departing from (44) and taking the derivatives shows that the income share of the technology providers increases when there is high elasticity of substitution between AI and labour, as can be seen from:

$$\frac{d\left(w_{IT}L_{IT}/Y\right)}{dA_{IT}}\frac{1}{w_{IT}L_{IT}/Y} = -\frac{\eta_{\phi_L,A_{IT}}}{\left(\frac{1}{\phi_L}-1\right)}\frac{dA_{IT}}{A_{IT}} > 0 \quad \text{with } \eta_{\phi_L,A_{IT}} < 0 \text{ for } 1 < \sigma \tag{75}$$

Income share of financial wealth holders: According to (45)the income share of financial wealth holders is $\frac{\alpha}{(1+\alpha)}$. Hence this income share will not change with the technology shock of AI progress, $dA_{IT} > 0$.

4.4 Impact on demand and absorption

Impact on consumption rate: From the above it is clear that a high elasticity of substitution leads to a shift in income distribution in favor of technology providers and financial wealth holders. Combining this insight with the result from section 3.5 that consumption demand is determined by the labor share, the consumption rate c is affected by progress in AI (dA_{IT}) as follows:[27]

$$\frac{dc}{c} = \overset{(-)}{\eta}_{c,A_{IT}}\frac{dA_{IT}}{A_{IT}} < 0 \tag{76}$$

with $\eta_{c,A_{IT}} = \eta_{\phi_L,A_{IT}} < 0$, for $\sigma > 1$. This shows that AI unambiguously tightens the demand constraint when the elasticity of substitution between AI technologies and labor is high ($\sigma > 1$).

[27]For a proof and the respective conditions see Appendix H.

4.5 Impact on long-term efficiency

How high current equilibrium output is compared to potential output, is measured by the utilization or deployment rate[28] which is $\omega = \tilde{Y}/\tilde{Y}^P$. $1-\omega$ is a of measure of (hidden) inefficiency. The economy is below its potential but the various agents are not aware of it. Given current equilibrium output (65) and potential output [maximum possible output $(\tilde{H}_Q = H)$, $\tilde{Y}^P(t) = N(t)(1-\alpha^2) H \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}$] the *deployment rate* is

$$\omega(t) = \frac{\tilde{Y}(t)}{\tilde{Y}^P(t)} = \frac{\tilde{H}_Q(t)}{H}. \tag{77}$$

AI progress dA_{IT} will affect $\omega(t)$ through the total availability of human services H and H_Q (see 62). As the effects on H and H_Q are different, the effect on the deployment rate, i.e. $\frac{d\omega(t)}{\omega(t)} = \frac{dH_Q}{H_Q} - \frac{dH}{H}$ is not clear. While $\frac{dH}{H}$ is known from (71) we need to determine the equilibrium change of human service in production, as result of AI progress which is given by[29]

$$\frac{d\tilde{H}_Q}{H_Q} = \eta_{H_Q,A_{IT}} \frac{dA_{IT}}{A_{IT}} > 0 \tag{78}$$

$$\eta_{H_Q,A_{IT}} = \frac{\frac{\phi_L}{(1+\alpha)}\left[\overset{<0}{\eta_{\phi_L,A_{IT}}} H_\phi + \overset{>0}{\eta_{H,A_{IT}}} H\right]}{[(2-\varphi)\, cH_Q + (1-\varphi)(-cov(\lambda,\varepsilon_F) - cH)]} > 0 \tag{79}$$

At a first glance the total effect $\frac{d\tilde{H}_Q}{H_Q}$ is ambiguous. There are two opposing forces. First, on the supply side, an increase in technology, which is quasi factor-augmenting, should lead to more factors available for production $\eta_{H,A_{IT}}$ in (78). Secondly, the term $\eta_{\phi_L,A_{IT}} < 0$ in (78) shows that the potential increase on the supply side is countered by a negative effect through income distribution and a reduction in absorption on the demand side. Higher inequality and a declining consumption rate restrict the total effect of AI on factor utilization for

[28] As a reminder, $\tilde{\omega}$ is the result of optimal individual behavior in steady state.

[29] See appendix H.

production which otherwise had been solely $\eta_{H,A_{IT}}$ in (78). The demand constraint can substantially reduce the supply side expansion. However, the overall effect of progress in AI remains positive as long as we do not assume additional effects of AI which may occur and reduce demand further. We discuss these additional affects in section 5.

After the identification of $\eta_{H_Q,A_{IT}}$ we can turn back to the deployment rate. If we plug in and rearrange, for the change of the deployment rate we obtain for a broad range of parameter values that

$$\frac{d\omega(t)}{\omega(t)} = \eta_{\omega,A_{IT}}\frac{dA_{IT}}{A_{IT}} < 0 \quad \text{for } \frac{(2-\varphi)}{(1-\varphi)}cH_{\phi} < -cov(\lambda,\varepsilon_F) \tag{80}$$

$$\eta_{\omega,A_{IT}} = \left[\frac{\frac{\phi_L}{(1+\alpha)}\left(\overset{<0}{\eta_{\phi_L,A_{IT}}}H_{\phi} + H\overset{>0}{\eta_{H,A_{IT}}}\right)}{(2-\varphi)cH_Q + (1-\varphi)(-cov(\lambda,\varepsilon_F) - cH)} - \overset{>0}{\eta_{H,A_{IT}}}\right] < 0 \tag{81}$$

If the co-variance $cov(\lambda,\varepsilon_F)$ is sufficiently large in absolute terms $\omega(t)$ decreases, such that an increase in A_{IT} increases the gap between current equilibrium steady state and potential steady state output.[30] In this comparative static analysis a once-and-for-all increase in AI technologies may lead to a positive or negative effect on deployment rate $\omega(t)$. However, from (74) and (76) we know that inequality increases and absorption declines if AI progress. As a consequence, with higher inequality and declining consumption, the deployment rate will decline and the economy will find itself on a long-term path of stagnating growth.

This path of stagnating growth has a simple intuitive explanation. If progress in AI technologies generate asymmetric benefits in favor of financial wealth holders and the owners of the AI technology, at the cost of the labor share of income and the consumption rate, the demand side will grow less than the supply side. Since our model has jettisoned Say's Law and market adjustments takes place through search and sales promotion decisions, the demand side can become a constraint on growth. Resources that could be used for more

[30]See Appendix H.

production and absorption are increasingly used to achieve a match between supply and demand. The economy could produce more, but has to deploy more and more resources to find the equilibrium.

4.6 Impact on wages and labor productivity

Changes in the expected ratio of market absorption $\frac{dc}{c}$ have implications for wages, labor productivity, and GDP growth. Because wages are equal to marginal labor productivity, the effects of AI on wage and labor productivity growth are:

$$\frac{dw_L(t)}{w_L(t)} = \left[\overset{(ii) \quad <0}{\eta_{\phi_L, A_{IT}}} + \overset{(i) \quad >0}{\eta_{H_Q, A_{IT}}} \right] \frac{dA_{IT}}{A_{IT}} \lesseqgtr 0 \tag{82}$$

The result in (82) shows that there are again two effects of AI on labor productivity and wages. First, growth in human service input and more human services in production is driven by IT and AI growth and described by $\eta_{H_Q, A_{IT}}$ [see (i) in 82 and (78)]. Second, if the elasticity of substitution is high ($\sigma > 1$) labor share of income and the consumption rate decline, i.e. $\eta_{\phi_L, A_{IT}} < 0$. This demand-constraining effect occurs because aggregate demand is not growing sufficiently to absorb all additional supply [see (ii) in 82]. Even more, if the co-variance $cov\,(\lambda, \varepsilon_F)$ is sufficiently large in absolute terms wage and labor productivity growth may stagnate or even decline.[31]

4.7 Impact on long-term GDP growth

The implementation of IT and AI technologies affect not only wages and labor, but also the GDP growth rate. The same mechanisms that were discussed in section 4.5 are also responsible for a negative net impact on GDP growth. These mechanisms are again (i)

[31] See Appendix H.

a positive productivity effect and (ii) a negative effect of a tightening demand constraint. Overall, however, the net impact is ambiguous. From (66) we know that the GDP growth rate is $\tilde{g}_Y = \left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi}{1-\alpha}} \left(\tilde{H}_Q\right)^{\varphi} (g_A)^{1-\varphi}$. Taking the derivative

$$\frac{d\tilde{g}_Y}{\tilde{g}_Y} = \varphi\eta_{H_Q,A_{IT}} \frac{dA_{IT}}{A_{IT}} > 0 \quad \text{with } \eta_{H_Q,A_{IT}} > 0 \text{ see (78)} \tag{83}$$

we see that the direction and extend of this effect depends on $\eta_{H_Q,A_{IT}}$. As section 4.5 provides an extensive discussion of equation (78), we can conclude: If A_{IT} increases, the overall growth rate increases as well. However, demand side effects restrict the growth expansion to a lower than potential level.

5 Effects with simultaneous AI-related demand shocks

Simultaneous demand shock: AI progress is often modelled or described as essentially process innovations - for instance the presentation of algorithms can learn in an unsupervised manner from data to process natural language. Think for example of advances in Machine Learning (ML) or Natural Language Processing (NLP). In the model presented in this paper we have however AI progress showing up in *product innovations*, specifically in the form of new intermediate products which are brought to the market following start-up investments by entrepreneurs. These intermediate products could be seen as embodying ML or NLP. Think for instance of a cobot that assists online shoppers. If AI progress changes the characteristics of such innovative intermediate goods and reduces start-up investment expenditures - as we found for IT and computer equipment - then v in (48) declines, specifically, $\frac{dv}{dA_{IT}} < 0$. In this case the impact of AI on the economy will be a combined, simultaneous shock on both the supply and the demand sides. In terms of GDP growth ($\tilde{g}_Y$) we now have that

$$
\begin{aligned}
\frac{d\tilde{g}_Y}{\tilde{g}_Y} &= \eta_{g_N,A_{IT}} \frac{dA_{IT}}{A_{IT}} \lesseqgtr 0 \quad \text{with} \\
\eta_{g_N,A_{IT}} &= \varphi \left[\frac{c \left[\eta_{c,A_{IT}} H_\phi + \eta_{H,A_{IT}} H \right] \frac{A_{IT}}{A_{IT}} - \frac{cov(\lambda,\varepsilon_F)}{(1-\alpha^2)} v \left(\frac{\alpha^2}{c_x} \right)^{\frac{\varphi-\alpha}{1-\alpha}} \left(\frac{g_A}{H_Q} \right)^{1-\varphi} \eta_{v,A_{IT}}}{\left[(2-\varphi) c H_Q + (1-\varphi) \left(-cov\left(\lambda,\varepsilon_F\right) - cH \right) \right]} \right] < 0 \\
& \quad \text{for sufficiently large } \left| cov\left(\lambda,\varepsilon_F\right) \right|, \text{ see appendix I.}
\end{aligned}
$$

Total production and the overall growth rate can turn negative if in absolute terms $cov(\lambda, \varepsilon_F)$ is sufficiently large. This will be the case if a positive (but potential weak) supply-side effect on process innovation is overcompensated by a simultaneous sufficiently large demand-side shock triggered by AI.

Simultaneous innovation shock: A second example illustrates, perhaps more forcefully, that progress in AI can lead to perverse innovation effects, particularly if it crowds out product innovations, for instance innovations in the intermediate goods sector. If AI progress does not fall from heaven, but is subject to opportunity cost, then in our model an increase in AI process innovations will lead to a reallocation of innovation away from the intermediate product sector. This effect could counter the positive AI resource shock. AI only becomes available through reallocation or reources and less intermediate innovation. Even if $dAI > 0$ is positive, the growth rate of innovation g_A may decline. With the simultaneous demand side effect of AI, total growth will be reduced as can be seen from the fact that

$$
\begin{aligned}
\frac{d\tilde{g}_Y}{\tilde{g}_Y} &= \eta_{g_N,A_{IT}} \frac{dA_{IT}}{A_{IT}} \lesseqgtr 0 \quad \text{with} \\
\eta_{g_N,A_{IT}} &= \left[\varphi \overset{<0}{\eta_{H_Q,A_{IT}}} + (1-\varphi) \overset{<0}{\eta_{g_A,A_{IT}}} \right] < 0, \\
& \quad \text{for sufficiently large } \left| cov\left(\lambda,\varepsilon_F\right) \right|, \text{ see appendix I.}
\end{aligned}
$$

This example describes a potential perverse innovation process where more resources are allocated towards process innovation and away from product innovation. As result then of a simultaneous demand and supply shocks, income and growth could decline even in absolute terms.

6 Concluding Remarks

In this paper we provided a semi-endogenous growth model that addressed two shortcomings of the growth model proposed by Acemoglu and Restrepo (2018b) - the AR-model. The first is that its reinstatement effects will depend, over the long-run, on the impact of AI automation on income distribution. If income inequality worsens, such as that the labor share in GDP declines, aggregate demand will decline. This would reduce the economy's actual and potential growth. Lower growth in turn would limit the reinstatement of new jobs. Unfortunately, the AR-model cannot take this into account, as it is supply-driven. We addressed this shortcoming by allowing for growth in our model to be demand constrained by replacing the typical assumption of a representative household by the assumption of two groups of households with different preferences.

The second shortcoming shortcoming of the AR-model, which is also a more general shortcoming of the task-approach on which it is based, is that it inadequately engages with the nature of AI and its technological feasibility. AI may perform tasks or help to perform tasks, but this depends on the abilities of AI and the abilities of labor. We addressed this shortcoming by adjusting the naive task-approach to incorporate AI as providing abilities. We modelled these abilities as the result of a combination of IT specific labor, IT specific abilities, and ML algorithms.

By integrating the task-based approach with a more nuanced specification of the nature of

AI and its technological feasibility in the form of abilities, we showed that AI automation can decrease the share of labor income evn for an substitution between AI and labor below one, and increase the income share of financial wealth owners and the owners of the technology. We also showed that when the elasticity of substitution between AI technologies and labor is high, AI will unambiguously reduce the aggregate consumption rate and retard aggregate demand expansion. With higher inequality and a declining consumption rate, the economy will move towards a declining utilization (deployment) rate of production potentials and increasing structural inefficiency.

Since our model has jettisoned the typical assumption of Say's Law - most often motivated by perfect price adjustments, and instead model market adjustments through search, sales promotion decisions and matching mechanisms, the demand side can become a binding constraint on the supply side. Resources that could be used for more production and absorption are used to match supply and demand. As a result the growth potential from AI is reduced.

Further, while progress in AI technologies would generate additional growth, this growth may turn negative if innovation activities and resources for AI simultaneously reduce innovation in traditional, non-AI fields. In our model progress in AI technologies amount to process innovations, and traditional innovations are product innovations. If AI process-innovations displace traditional product innovations, economic growth may turn negative. Furthermore, wages can stagnate in line with slower GDP and productivity growth so as to maintain employment levels. Thus, in the model presented here we can explain why contemporary advanced countries experience the simultaneous existence of high employment with stagnating wages, productivity and GDP, all in spite of AI-progress. Hence, there is no race *between* man and machine, only a race *of* man and machine.

Appendices

A. Final-goods-producing firm

Implicit Function Theorem for optimal $H_{\phi i}$ Condition for applying the implicit function theorem hold: $0 = F = E\left[\phi_i\left(\delta_i, H_{\phi i}\right)\right] - 1$, and $\frac{dF}{dH_{\phi i}} = \frac{\partial E[\phi_i(t)]}{\partial H_{\phi i}(t)} > 0$. For the effect of $E\left[\delta_i\right]$ we use $\frac{dF}{dE[\delta_i]} = \frac{\partial E[\phi_i(t)]}{\partial E[\delta_i]} < 0$.

B. The task-based approach

B1. The optimal allocation of tasks, and task production

Demand for tasks: Human service firms

$$\max : \pi_H = p_H H - p_h(z)h(z) = p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} - p_h(z)h(z).$$

F.O.C.

$$\begin{aligned} p_H \frac{\sigma}{\sigma-1} \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} h(z)^{\frac{\sigma-1}{\sigma}-1} - p_h(z) = 0 \\ p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}-1} h(z)^{\frac{\sigma-1}{\sigma}-1} = p_h(z) \\ p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{1}{\sigma-1}} h(z)^{-\frac{1}{\sigma}} = p_h(z) \\ p_H H^{\frac{1}{\sigma}} h(z)^{-\frac{1}{\sigma}} = p_h(z) \end{aligned}$$

arriving at $h(z) = \frac{H}{p_h(z)^{\sigma}} p_H^{\sigma}$, see (13).

Demand for task z **:** Using marginal production and productivity rules

$h(z_{IT}) = A_{IT}\gamma_{IT}(z)l_{IT}(z)$	production (10)	$h(z_L) = A_L\gamma_L(z)l_L(z)$
$p_h A_{IT}\gamma_{IT}(z)l_{IT}(z) = l_{IT}(z)w_{IT}$	marginal productivity and factor reward	$p_h A_L\gamma_L(z)l_L(z) = l_L(z)w_L$
$p_h\left(z_{IT}\right) = \frac{w_{IT}}{A_{IT}\gamma_{IT}(z_{IT})}$	price = unit labor costs	$p_h\left(z_L\right) = \frac{w_L}{A_L\gamma_L(z_L)}$

and plugging in gives (14) as being the optimal demand for $h(z)$,

$$\begin{array}{ll} h(z) = \frac{H}{\left(\frac{w_{IT}}{A_{IT}\gamma_{IT}(z)}\right)^{\sigma}} p_H^{\sigma}, & h(z) = \frac{H}{\left(\frac{w_L}{A_L\gamma_L(z)}\right)^{\sigma}} p_H^{\sigma}, \\ h(z) = p_H^{\sigma} H\left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\gamma_{IT}(z)^{\sigma}, & h(z) = p_H^{\sigma} H\left(\frac{A_L}{w_L}\right)^{\sigma}\gamma_L(z)^{\sigma}. \end{array}$$

Demand for various kinds of labor: In order to determine the marginal productivity for each total labor input, the productivity for each kind of labor is derived from (14) and (10), and we can obtain the optimal demand for IT labor :

$$\begin{aligned} h(z) &= p_H^{\sigma} H\left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\gamma_{IT}(z)^{\sigma} \\ A_{IT}\gamma_{IT}(z)l_{IT}(z) &= p_H^{\sigma} H\left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\gamma_{IT}(z)^{\sigma} \\ l_{IT}(z) &= p_H^{\sigma} H\left(A_{IT}\right)^{\sigma-1} w_{IT}^{-\sigma}\gamma_{IT}(z)^{\sigma-1}, \text{ see (15)}, \end{aligned}$$

and standard labor:

$$l_L(z) = p_H^{\sigma} H\left(A_L\right)^{\sigma-1} w_L^{-\sigma}\gamma_L(z)^{\sigma-1}, \text{ see (16)}.$$

To determine wages for each kind of labor we have to rearrange. As the following calculations are symmetric for each kind of labor, we present the details only for L_{IT}

$$l_{IT}(z) = p_H^{\sigma} H\left(A_{IT}\right)^{\sigma-1} w_{IT}^{-\sigma}\gamma_{IT}(z)^{\sigma-1}$$

Total IT labor is fully employed and allocated to all tasks using IT labor.

$$L_{IT} = \int_{N-1}^{N_{IT}} l_{IT}(z)dz.$$

With the integral in (15) $\left[l_{IT}(z)=\frac{p_H^\sigma}{w_{IT}^\sigma}H\gamma_{IT}(z)^{\sigma-1}\left(A_{IT}\right)^{\sigma-1}\right]$ we obtain

$$\begin{aligned}\int_{N-1}^{N_{IT}} l_{IT}(z)dz &= \int_{I}^{N_{IT}} \frac{p_H^\sigma}{w_{IT}^\sigma}H\gamma_{IT}(z)^{\sigma-1}\left(A_{IT}\right)^{\sigma-1}dz \\ L_{IT} &= \frac{p_H^\sigma}{w_{IT}^\sigma}H\left(A_{IT}\right)^{\sigma-1}\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz \\ w_{IT}^\sigma &= p_H^\sigma\frac{H}{L_{IT}}\left(A_{IT}\right)^{\sigma-1}\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\end{aligned}$$

such that with fully employed IT labor we can determine their wages as

$$w_{IT}=p_H\left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}}\left(A_{IT}\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}, \tag{84}$$

and, in a symmetrical fashion we obtain for standard labor

$$w_L=p_H\left(\frac{H}{L_L}\right)^{\frac{1}{\sigma}}\left(A_L\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}. \tag{85}$$

The resulting internal relative factor productivity for labor is:

$$\begin{aligned}\frac{w_L}{w_{IT}} &= \frac{\left(\frac{p_H H}{L_L}\right)^{\frac{1}{\sigma}}\left(A_L\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}}{\left(\frac{p_H H}{L_{IT}}\right)^{\frac{1}{\sigma}}\left(A_{IT}\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}} \\ \frac{w_L}{w_{IT}} &= \left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}}\end{aligned}$$

Endogenously automated tasks N_{IT} **:** From the discussion of (11) it is known that tasks are ordered such that $\gamma(z)=\frac{\gamma_L(z)}{\gamma_{IT}(z)}$, and $\frac{\partial\gamma(z)}{\partial z}>0$. If it is assumed that task N_{IT} is the task that exactly separates the production mode, and if tasks are continues, the condition (11) can be rewritten as follows:

$$\begin{aligned}\frac{A_L\gamma_L(N_{IT})}{A_{IT}\gamma_{IT}(N_{IT})} &< \frac{w_L}{w_{IT}}=\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}}\left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \\ 0 &= G=\gamma\left(N_{IT}\right)-\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}}\end{aligned} \tag{86}$$

If $\frac{dG}{dN_{IT}} \neq 0$, G implicitly defines a function $N_{IT} = N_{IT}\left(L_{IT}, L_L, A_{IT}, ...\right)$.Thus, we need to calculate the respective interesting derivatives.

$$\frac{dG}{dN_{IT}} = \frac{\partial \gamma\left(N_{IT}\right)}{\partial N_{IT}} + \left[\begin{array}{c} \frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{I}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \\ \left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{\int_{N_{IT}}^{N}\gamma_L(N_{IT})^{\sigma-1}dz} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{\int_{N-1}^{N_{IT}}\gamma_{IT}(N_{IT})^{\sigma-1}dz}\right] \end{array} \right] > 0$$

and defining $B_1\left(N_{IT}\right) = \int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz = (1-\Gamma(N_{IT}, N))\Pi(N_{IT}, N)^{\sigma-1}$, $\frac{dB_1}{dN_{IT}} = \gamma_{IT}(N_{IT})^{\sigma-1}$; and $B_2\left(N_{IT}\right) = \int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$, $\frac{dB_2}{dN_{IT}} = -\gamma_L(N_{IT})^{\sigma-1}$ we obtain

$$\begin{aligned}
\frac{\partial G}{\partial N_{IT}} &= \frac{\partial \gamma\left(N_{IT}\right)}{\partial N_{IT}} + \frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{B_2\left(N_{IT}\right)}{B_1\left(N_{IT}\right)}\right)^{\frac{1}{\sigma}}\left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2\left(N_{IT}\right)} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1\left(N_{IT}\right)}\right] > 0 \\
\frac{\partial G}{\partial A_{IT}} &= -\frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}-1}\left(\frac{B_2\left(N_{IT}\right)}{B_1\left(N_{IT}\right)}\right)^{\frac{1}{\sigma}}\frac{L_{IT}}{A_L L_L} < 0
\end{aligned}$$

and the derivative of the implicit function $N_{IT} = N_{IT}\left(A_{IT}\right)$ is

$$\frac{dN_{IT}}{dA_{IT}} = -\frac{\frac{\partial G}{\partial A_{IT}}}{\frac{\partial G}{\partial N_{IT}}} > 0$$

More specifically:

$$\begin{aligned}
\frac{dN_{IT}}{dA_{IT}} &= \frac{\frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}}\frac{1}{A_{IT}}}{\frac{\partial\gamma(N_{IT})}{\partial N_{IT}} + \frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}}\left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})}\right]} \\
\eta_{N_{IT},A_{IT}} &= \frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}} = \frac{1}{\sigma\frac{\partial\gamma(N_{IT})}{\partial N_{IT}}\left(\frac{A_L L_L}{A_{IT}L_{IT}}\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}} + \frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})}}\frac{1}{N_{IT}}
\end{aligned}$$

B2. Total supply of human service inputs

From (14) it is known that $h(z) = p_H^{\sigma}H\left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\gamma_{IT}(z)^{\sigma}$ for $z \in [N-1, N_{IT}.$, and $h(z) = p_H^{\sigma}H\left(\frac{A_L}{w_L}\right)^{\sigma}\gamma_L(z)^{\sigma}$ for $z \in [N_{IT}, N]$. Plugging this in (9) generates an expression for the total value of H:

$$\begin{aligned} H &= \left(\int_{N-1}^{N_{IT}} h(z)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_{N-1}^{N_{IT}} \left(p_H^\sigma H \left(\frac{A_{IT}}{w_{IT}}\right)^\sigma \gamma_I(z)^\sigma\right)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} \left(p_H^\sigma H \left(\frac{A_L}{w_L}\right)^\sigma \gamma_L(z)^\sigma\right)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}. \end{aligned}$$

Using (84) and (85) results in: $w_{IT} = p_H \left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}} (A_{IT})^{\frac{\sigma-1}{\sigma}} \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}$

$$\begin{aligned} H &= \left(\int_{N-1}^{N_{IT}} (\gamma_{IT}(z)^\sigma)^{\frac{\sigma-1}{\sigma}} dz \left(p_H^\sigma H \left(\frac{A_{IT}}{w_{IT}}\right)^\sigma\right)^{\frac{\sigma-1}{\sigma}} + \int_{N_{IT}}^{N} (\gamma_L(z)^\sigma)^{\frac{\sigma-1}{\sigma}} dz \left(p_H^\sigma H \left(\frac{A_L}{w_L}\right)^\sigma\right)^{\frac{\sigma-1}{\sigma}}\right) \quad (87) \\ &= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{w_L}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

$$= \left(\begin{array}{l} \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{p_H \left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}} (A_{IT})^{\frac{\sigma-1}{\sigma}} \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \\ + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{p_H \left(\frac{H}{L_L}\right)^{\frac{1}{\sigma}} (A_L)^{\frac{\sigma-1}{\sigma}} \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \end{array}\right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\begin{array}{l} \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{p_H^{-1} H^{-\frac{1}{\sigma}} L_{IT}^{\frac{1}{\sigma}} A_{IT}^{\frac{1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \\ + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{p_H^{-1} H^{-\frac{1}{\sigma}} L_L^{\frac{1}{\sigma}} A_L^{\frac{1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \end{array}\right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\begin{array}{l} \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \frac{p_H^{-(\sigma-1)} H^{-\frac{\sigma-1}{\sigma}} (L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} \\ + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \frac{p_H^{-(\sigma-1)} H^{-\frac{\sigma-1}{\sigma}} (L_L A_L)^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} \end{array}\right)^{\frac{\sigma}{\sigma-1}}$$

$$= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \frac{(L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} d \frac{(L_L A_L)^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$H = \left(\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}} (L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}} (L_L A_L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

B2.1 Earning shares

To determine the contribution of standard labor to total service production one can start from (87)

$$H = \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{w_{IT}} \right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{w_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned}
1 &= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \left(\frac{A_{IT}}{w_{IT}} \right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \left(\frac{A_L}{w_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} p_H^{\sigma} \\
1 &= \left((1-\Gamma(N_{IT},N))\Pi(N_{IT},N)^{\sigma-1} \left(\frac{A_{IT}}{w_{IT}} \right)^{\sigma-1} + \Gamma(N_{IT},N)\Pi(N_{IT},N)^{\sigma-1} \left(\frac{A_L}{w_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} p_H^{\sigma}
\end{aligned}$$

Plugging in definitions (22) and (21),$\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz = \Gamma(N_{IT},N)\Pi(N_{IT},N)^{\sigma-1}$, $\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz = (1-\Gamma(N_{IT},N))\Pi(N_{IT},N)^{\sigma-1}$ we obtain

$$\begin{aligned}
1 &= \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \frac{\left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma-1}}{\left(\frac{A_L}{w_L}\right)^{\sigma-1}} + 1 \right)^{\frac{\sigma}{\sigma-1}} \Gamma(N_{IT},N)\Pi(N_{IT},N)^{\sigma} \left(\frac{A_L}{w_L} \right)^{\sigma} p_H^{\sigma} \\
1 &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} \Gamma(N_{IT},N)^{\frac{\sigma}{\sigma-1}} \Pi(N_{IT},N)^{\sigma} \left(\frac{A_L}{w_L} \right)^{\sigma} p_H^{\sigma}
\end{aligned}$$

Rearrange this equation gives:

$$\begin{aligned}
\Gamma(N_{IT},N)^{-\frac{\sigma}{\sigma-1}} \Pi(N_{IT},N)^{-\sigma} &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{A_L}{w_L} \right)^{\sigma} p_H^{\sigma} \\
\Pi(N_{IT},N)^{\sigma-1}\Gamma(N_{IT},N) &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L} \right)^{\sigma-1} \right)^{-1} (A_L)^{-(\sigma-1)} \left(\frac{p_H}{w_L} \right)^{-(\sigma-1)} \quad (88) \\
(A_L)^{(\sigma-1)} \left(\frac{p_H}{w_L} \right)^{(\sigma-1)} &= \frac{1}{\left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L} \right)^{\sigma-1} \right) \Pi(N_{IT},N)^{\sigma-1}\Gamma(N_{IT},N)}
\end{aligned}$$

Further, from definition (18) and (16) the following expression can be derived:

$$L_L = \int_{N_{IT}}^{N} \frac{p_H^{\sigma} H}{(w_L)^{\sigma}} (A_L)^{\sigma-1} \gamma_L(z)^{\sigma-1} dz = \frac{p_H^{\sigma} H}{(w_L)^{\sigma}} (A_L)^{\sigma-1} \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz$$

using the definition of labor share of income $\phi_L = \frac{w_L L}{p_H H}$ and using the definitions (22) and (21), $\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$ results in:

$$\begin{aligned} \frac{L_L w_L}{p_H H} &= \frac{w_L}{p_H} \frac{1}{H} \frac{p_H^{\sigma} H}{(w_L)^{\sigma}} (A_L)^{\sigma-1} \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \\ \frac{L_L w_L}{p_H H} &= \left(\frac{w_L}{p_H}\right)^{1-\sigma} (A_L)^{\sigma-1} \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1} \end{aligned}$$

Combining these with (88) gives labor's share of income as fully depending on relative labor rewards $\frac{w_L}{w_{IT}}$

$$\begin{aligned} \phi_L &= \frac{\Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}}{\left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right) \Pi(N_{IT}, N)^{\sigma-1}\Gamma(N_{IT}, N)} \\ \phi_L &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{w_L}{w_{IT}} \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{-1} \end{aligned}$$

Labor's share of income in the human service sector is determined by relative factor abundance and productivity parameters. Thus, plugging in the relative factor rewards (19) finally results in:

$$\begin{aligned} \phi_L &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\left[\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}} \left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\Gamma(N_{IT},N)}{(1-\Gamma(N_{IT},N))}\right)^{\frac{1}{\sigma}}\right] \frac{A_{IT}}{A_L}\right)^{\sigma-1}\right)^{-1} \\ &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}} \left(\frac{A_{IT}}{A_L}\right)^{\frac{1}{\sigma}} \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{-\frac{1}{\sigma}}\right)^{\sigma-1}\right)^{-1} \\ &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{A_{IT}}{A_L}\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{-\frac{\sigma-1}{\sigma}}\right)^{-1} \\ &= \left(1 + \frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)} \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{-\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{A_L}\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1} \\ &= \left(1 + \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{\frac{1}{\sigma}} \left(\frac{A_{IT}}{A_L}\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1} \end{aligned}$$

Which results in expression (24).

C. Income distribution

Income Y (GDP) and total production Q. Before determining the income shares it is necessary to determine the relation between income Y (GDP) and total production Q. According to (38)

$$Y(t) = Q(t) - N(t)x(t)c_x = \left(1 - \frac{Nxc_x}{Q}\right) Q(t).$$

Applying (27) gives

$$\frac{Nxc_x}{Q} = \frac{Nc_x}{NH_Q^{1-\alpha}\alpha^{\frac{-(1-\alpha)2}{1-\alpha}} c_x^{-\frac{-(1-\alpha)}{1-\alpha}} H_Q^{-(1-\alpha)}} = \frac{c_x}{\alpha^{-2}c_x} = \alpha^2,$$

and for $Y(t)$ the result is:

$$Y(t) = \left(1 - \alpha^2\right) Q(t) = \left(1 - \alpha^2\right) N(t)\alpha^{\frac{2\alpha}{1-\alpha}} c_x^{-\frac{\alpha}{1-\alpha}} H_Q \tag{89}$$

Income share of labor: If A_L is time depending)(i.e. $A_L(t)$) and continuously increasing the long-term position is:

$$\begin{aligned} \frac{w_L L_L}{Y} &= \frac{(1-\alpha)\phi_L}{1-\alpha^2} = \frac{\phi_L}{1+\alpha} \\ \lim_{A_{IT}\to\infty} \frac{w_L L_L}{Y} &= \frac{\phi_L}{1+\alpha} = \lim_{A_{IT}\to\infty} \phi_L = \frac{1}{1 + \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{\frac{1}{\sigma}} \left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}} = 0 \quad \text{for} \quad \sigma > 1. \end{aligned}$$

Wages and income share of IT providers:

$$\frac{w_{IT}(t) L_{IT}}{Y(t)} = \frac{(1-\alpha)(1-\phi_L)}{1-\alpha^2} = \frac{1-\phi_L}{1+\alpha}$$

Income share of financial wealth owners: From (25), (27) an (38) it can be seen that:

$$\begin{aligned}
\frac{N(t)\,\pi_x(t)}{Y(t)} &= \frac{N\left(\frac{1}{\alpha}-1\right)(c_x)\left[\alpha^{\frac{2}{1-\alpha}}c_x^{-\frac{1}{1-\alpha}}H_Q\right]}{NH_Q^{1-\alpha}x^{\alpha}-Nxc_x} \\
&= \frac{\left(\frac{1}{\alpha}-1\right)Nc_x x}{NH_Q^{1-\alpha}x^{\alpha}-Nxc_x} \\
&= \frac{N\left(\frac{1}{\alpha}-1\right)c_x x}{\left(1-\frac{Nxc_x}{Q}\right)NH_Q^{1-\alpha}x^{\alpha}}
\end{aligned}$$

Using $\frac{Nxc_x}{Q}=\alpha^2$ results in:

$$\frac{N(t)\,\pi_x(t)}{Y(t)} = \frac{(1-\alpha)\,\frac{1}{\alpha}\alpha^2}{(1-\alpha^2)} = \frac{(\alpha-\alpha^2)}{(1-\alpha^2)} = \frac{\alpha}{(1+\alpha)}.$$

D. Inter-temporal choices for labor and capital owners

In standard endogenous growth models, aggregate consumption expenditure and savings are determined by a representative household conducting an optimal intertemporal choice according to the Euler equation

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\eta_U}.$$

However, this assumption of a *representative household* is rather restrictive and is introduced more for the sake of simplification. Therefore, in the model proposed in this paper, this assumption is replaced by assuming two groups of households differing with respect to intertemporal choice behaviour.

(i) It is assumed that workers with wage income represent the low income group. The second group, the owners of financial assets F, represents the high income group. For these households returns r are the only source of income. (ii) Households in each group make their own intertemporal choices. Both ρ and η_U vary across low- and high-income households. a) *Low-income, wage-earning households*: If it is assumed that the time preference rate of low-income households is high, e.g. $\rho_L \geq r$, and if household debt is not allowed, then the Euler equation $\frac{\dot{C}_L}{C_L} = \frac{r-\rho_L}{\eta_{U_L}}$ implies that these households do not intend to shift intertemporal consumption and simply consume what they earn from wage income. b) *High-income households*: High-income households obtain their total income from returns on financial assets

$F(t)\,r$.

Thus, the budget constraint of high-income, financial asset owners is $S_F(t) \leq F(t)\,r - C_F(t)$. As savings are used to purchase newly issued financial assets $\dot{F}(t)$ and these assets finance investments, we obtain $\dot{F}(t) = F(t)\,r - C_F(t)$, and

$$\frac{\dot{F}(t)}{F(t)} = \frac{F(t)}{F(t)}r - \frac{C_F(t)}{F(t)} \tag{A1}$$

Applying the Euler equation for financial investors $\frac{\dot{C}_F}{C_F} = \frac{r-\rho_F}{\eta_{U_F}}$ and using the fact that in long-term steady-state growth of consumption cannot exceed the economy's growth rate g_N, we obtain $g_N \geq \frac{\dot{C}_F}{C_F} = \frac{r-\rho_F}{\eta_{U_F}}$. However, this group's consumption growth remains below GDP growth if $g_N \geq \frac{r-\rho_F}{\eta_{U_F}}$, and this holds for values of η_{U_F} and ρ_F that satisfy $\eta_{U_F} \geq \frac{r-\rho_F}{g_N}$. Further, if $F(t)$ grows equal to the economy's rate of growth g_N we obtain for $\frac{C_F(t)}{F(t)} = \frac{C_F(t)}{N(t)} = \frac{C_F(0)e^{g_{C_F}t}}{N(0)e^{g_N t}}$ and thus $\lim\limits_{t\to\infty} \frac{C_F(0)}{N(0)} e^{-\left(g_N - g_{C_F}\right)t} = 0$.

This condition fits the requirement of dynamic consistency and allows us to determine the start value $\frac{F(0)}{N(0)}$.

When taking the limit, financial investors' consumption rate turns to zero in steady state, $\lim\limits_{t\to\infty} c_F = \frac{C_F(t)}{Y(t)} = \frac{C_F(t)}{N(t)(\lambda L^{1-\alpha}x^{\alpha} - x c_x)} = \frac{C_F(t)}{N(t)}\left(\tilde{\lambda}L^{1-\alpha}\tilde{x}^{\alpha} - \tilde{x}c_x\right)^{-1} = 0$, for steady state values $\tilde{\lambda}$ and $\tilde{x}$.

This illustrates the assumption that financial investors only save and do not consume - at least in the long term steady state.

E. Calculations to solve the model

Determine the growth rate g_N: From (33) and (27) we obtain $X^{eD} = Nx = N\alpha^{\frac{2}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H_Q$. and $\dot{N} = \left(\alpha^{\frac{2}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H_Q\right)^{\varphi} N^{\varphi}(\dot{A}))^{1-\varphi}$. Rearranging gives $\frac{\dot{N}(t)}{N(t)} = \left(\alpha^{\frac{2}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H_Q\right)^{\varphi} (\frac{\dot{A}(t)}{A(t)})^{1-\varphi}$ for $N(t) = A(t)$.

Firm's sales equilibrium: Combing (2) for the aggregate economy $\phi(t) = 1 - \delta(t)$ with $\delta(t) = \delta'(t) - m$ (see 57) leads to $\phi(t) = 1 - \delta'(t) + m$, and with the definition of the firms sales frction friction $\delta'(t) = (1-\lambda)\,\varepsilon_F$, and taking expectations gives

$$= 1 - E[\varepsilon_F] + E[\lambda]\,E[\varepsilon_F] + cov(\lambda, \varepsilon_F) + H_\phi (1-\theta)^{-1}$$

Thus, in sales equilibrium described by condition (59) $E[\phi(t)] \overset{!}{=} 1$, leads to

$$H_\phi (1-\theta)^{-1} = E[\varepsilon_F] - E[\lambda]\,E[\varepsilon_F] - cov(\lambda, \varepsilon_F).$$

Using (34) and (60) we obtain (59a)

Determine current aggregate market equilibrium: According to (60) market equilibrium requires $1 = E[\lambda(t)]$. With (51)

$$1 = E[\lambda(t)] = \frac{\nu}{1-c(1-\theta)} \frac{1}{H_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} g_N + \alpha^2,$$

and using (52) leads to

$$1 = \frac{\nu}{1-c(1-\theta)} \frac{1}{H_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} \left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q \right)^{\varphi} (g_A)^{1-\varphi} + \alpha^2.$$

and (60a)

F. Proof of proposition 1

Implicit Function theorem: Function F can be derived by:

$$\begin{aligned} H_\phi (1-\theta)^{-1} &= -cov(\lambda, \varepsilon_F). \\ (1-\theta) &= \frac{H - H_Q}{-cov(\lambda, \varepsilon_F)} \end{aligned}$$

and

$$
\begin{aligned}
&= \frac{\nu}{\left(1-\alpha^2\right)\left(1-c\left(1-\theta\right)\right)}\frac{1}{H_Q^{1-\varphi}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}\varphi}\left(g_A\right)^{1-\varphi} \\
\left(1-\theta\right) &= -\frac{\nu}{c\left(1-\alpha^2\right)}\frac{1}{H_Q^{1-\varphi}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}\varphi}\left(g_A\right)^{1-\varphi}+\frac{1}{c}
\end{aligned}
$$

plugging in gives

$$
\begin{aligned}
\frac{H-H_Q}{-cov\left(\lambda,\varepsilon_F\right)} &= -\frac{\nu}{c\left(1-\alpha^2\right)}\frac{1}{H_Q^{1-\varphi}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(g_A\right)^{1-\varphi}+\frac{1}{c} \\
\left(H-H_Q\right)H_Q^{1-\varphi}-\frac{-cov\left(\lambda,\varepsilon_F\right)H_Q^{1-\varphi}}{c} &= -\frac{-cov\left(\lambda,\varepsilon_F\right)\nu}{c\left(1-\alpha^2\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(g_A\right)^{1-\varphi}
\end{aligned}
$$

$$
\begin{aligned}
0 &= F=-c\left(H-H_Q\right)H_Q^{1-\varphi}-cov\left(\lambda,\varepsilon_F\right)H_Q^{1-\varphi}+\frac{-cov\left(\lambda,\varepsilon_F\right)\nu}{c\left(1-\alpha^2\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(g_A\right)^{1-\varphi} \\
F &= -cHH_Q^{1-\varphi}+cH_Q^{2-\varphi}-cov\left(\lambda,\varepsilon_F\right)H_Q^{1-\varphi}-\frac{-cov\left(\lambda,\varepsilon_F\right)\nu}{\left(1-\alpha^2\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(g_A\right)^{1-\varphi} \\
&= cH_Q^{2-\varphi}+\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)H_Q^{1-\varphi}-\frac{-cov\left(\lambda,\varepsilon_F\right)\nu}{\left(1-\alpha^2\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(g_A\right)^{1-\varphi}
\end{aligned}
$$

To apply the Implicit Function Theorem $\frac{dF}{dH_Q}\neq 0$, :

$$
\begin{aligned}
\frac{dF}{dH_Q} &= \left(2-\varphi\right)cH_Q^{1-\varphi}+\left(1-\varphi\right)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)H_Q^{-\varphi} \\
&= \left[\left(2-\varphi\right)cH_Q+\left(1-\varphi\right)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)\right]H_Q^{-\varphi}>0
\end{aligned}
$$

$$
\begin{aligned}
(2-\varphi)\,cH_Q - cov\,(\lambda,\varepsilon_F)\,(1-\varphi) - (1-\varphi)\,cH &> 0 \\
-cov\,(\lambda,\varepsilon_F) &> c\left(H - \frac{(2-\varphi)}{(1-\varphi)}H_Q\right) \\
\frac{(2-\varphi)\,H_Q}{(1-\varphi)} &> H \\
\frac{H_Q}{H} &> \frac{(1-\varphi)}{(2-\varphi)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dF}{dA_{IT}} &= -\frac{dc}{dA_{IT}}\left(HH_Q^{1-\varphi} - H_Q^{2-\varphi}\right) - c\frac{dH}{dA_{IT}}H_Q^{1-\varphi} \\
&= \left[-\frac{dc}{dA_{IT}}\left(H-H_Q\right) - c\frac{dH}{dA_{IT}}\right]H_Q^{1-\varphi} \\
&= \left[-\frac{dc}{dA_{IT}}\frac{A_{IT}}{c}\frac{dA_{IT}}{A_{IT}}c\left(H-H_Q\right) - c\frac{dH}{dA_{IT}}\frac{A_{IT}}{H}H\frac{1}{A_{IT}}\right]H_Q^{1-\varphi} \\
&= -c\left[\frac{dc}{dA_{IT}}\frac{A_{IT}}{c}\left(H-H_Q\right) + \frac{dH}{dA_{IT}}\frac{A_{IT}}{H}H\right]\frac{1}{A_{IT}}H_Q^{1-\varphi}
\end{aligned}
$$

$\frac{dH_Q}{dA_{IT}}$:

$$
\frac{dH_Q}{dA_{IT}} = -\frac{\frac{\partial F}{\partial A_{IT}}}{\frac{\partial F}{\partial \tilde{H}_Q}} = -\frac{-c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi}}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\,(\lambda,\varepsilon_F) - cH\right)\right]H_Q^{-\varphi}}
$$

$\frac{dH_Q}{dg_A}$:

$$
\frac{dF}{dg_A} = (1-\varphi)\,\frac{cov\,(\lambda,\varepsilon_F)\,\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{-\varphi} < 0
$$

$$
\frac{dH_Q}{dg_A} = -\frac{(1-\varphi)\,cov\,(\lambda,\varepsilon_F)\,\nu\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{-\varphi}}{(1-\alpha^2)\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\,(\lambda,\varepsilon_F) - cH\right)\right]H_Q^{-\varphi}} > 0
$$

$\frac{dH_Q}{dv}$:

$$
\frac{dF}{dv} = cov\,(\lambda,\varepsilon_F)\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{1-\varphi}
$$

$$
\frac{dH_Q}{dv} = -\frac{cov\,(\lambda,\varepsilon_F)\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{1-\varphi}}{(1-\alpha^2)\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\,(\lambda,\varepsilon_F) - cH\right)\right]H_Q^{-\varphi}} > 0
$$

With $cov\,(\lambda,\varepsilon_F) < 0$ the derivative $\frac{dF}{dH_Q} < 0$ and the implicit function theorem (requiring

$\frac{dF}{dL_Q} \neq 0$) can be applied.

q.e.d.

Other equilibrium values: From (59a) and (62) we dictly obtain (63). From Production function (1) and the optimal intermediate goods input (27) ($x(t) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q$) we obtain (64). Using (89) in 6 we obtain the Y. Combining with (64) gives (89). Taking the time derivative of (65) in equilibrium we obtain $\dot{Y}(t) = \dot{N}(t)(1-\alpha^2) H_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}$ and thus $g_Y = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{N}(t)}{N(t)}$, and using (52) we arrive at (66). According to (48) investments are $I(t) = \dot{N}(t)\nu$ and from (32) we know $r(t) = \frac{\pi_x(t)}{\nu}$. With all profits being saved we obtain $\dot{N}(t)\nu = I_x(t) = S(t) = N(t)\pi_x(t)$. Plugging in (32) gives (67).

G. Dynamic consistency

Consistent start values of financial and technology stocks: It can be shown that derived savings can finance the process from the start. According to the discussion in section 2.4 financial wealth income is $rF(t)$. As it is assumed that only labor income consumes, the income of financial asset holders and technology owners only serves for savings and these savings are financing investments for newly introduced goods. Two version of the budget constraint can be derived: one that describes the real investments and innovation which becomes possible ($S = I_x$) and the second that describes the finance mechanism ($S(t) = \dot{F}(t)$)

$$
\begin{aligned}
(i) &: \; N(t)\pi_x(t) + w_{IT}(t) L_{IT} = S(t) = I_x(t) = \dot{N}(t)\nu_x \\
(ii) &: \; rF(t) + w_{IT}(t) L_{IT} = S(t) = \dot{F}(t)
\end{aligned}
$$

Equation (43) describes factor rewards for technology owners. $w_{IT}(t) = \frac{1-\phi_L}{1+\alpha}\frac{Y(t)}{L_{IT}}$, with $Y(t) = (1-\alpha^2) Q(t) = (1-\alpha^2) N(t)\alpha^{\frac{2\alpha}{1-\alpha}} c_x^{-\frac{\alpha}{1-\alpha}} H_Q$ and $(1-\alpha^2) = (\alpha+1)(1-\alpha)$ the implication is that $w_{IT}(t) = N(t)(1-\phi_L)(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} c_x^{-\frac{\alpha}{1-\alpha}} H_Q \frac{1}{L_{IT}}$.
Defining $Z_{IT} = (1-\phi_L)(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} c_x^{-\frac{\alpha}{1-\alpha}} H_Q$, results in $w_{IT}(t) = N(t)\frac{Z_{IT}}{L_{IT}}$, and total income of technology owners is $w_{IT}(t) L_{IT} = N(t) Z_{IT}$.

Thus from (i) the implication is that:

$$\begin{aligned}
(i) \quad \dot{N}(t)\,\nu_x &= N(t)\,\pi_x(t) + N(t)\,Z_{IT} \\
\frac{\dot{N}(t)}{N(t)}\nu_x &= \pi_x(t) + zZ_{IT} \\
\frac{\dot{N}(t)}{N(t)} &= r + \frac{1}{\nu_x}Z_{IT}. \quad or \quad r = \frac{\dot{N}(t)}{N(t)} - \frac{Z_{IT}}{\nu_x}
\end{aligned}$$

From (ii) the implication is that:

$$\begin{aligned}
(ii) \quad \dot{F}(t) &= rF(t) + N(t)\,Z_{IT} \\
\frac{\dot{F}(t)}{F(t)} &= r + \frac{N(t)}{F(t)}Z_{IT}
\end{aligned}$$

plugging in from (i) $r = \frac{\dot{N}(t)}{N(t)} - \frac{1}{\nu_x}Z_{IT}$ into (ii) results in:

$$\begin{aligned}
\frac{\dot{F}(t)}{F(t)} &= g_N - \frac{1}{\nu_x}Z_{IT} + \frac{N(t)}{F(t)}Z_{IT} \\
g_F &= g_N + \left[\frac{N(0)\,e^{g_N}}{F(0)\,e^{g_F}} - \frac{1}{\nu_x}\right]Z_{IT}
\end{aligned}$$

and this holds if in steady state growth $g_N = g_F$ and $\frac{N(0)}{F(0)} = \frac{1}{\nu_x}$.

q.e.d.

H. Effects of automation

Effects on human service H:

Departing from (23) $H = \Pi(N_{IT}, N)\left[(1-\Gamma(N_{IT}, N))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} + \Gamma(N_{IT}, N)^{\frac{1}{\sigma}}(A_L L_L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ and thus

$$H = \left[\left((1-\Gamma(N_{IT}, N))\,\Pi(N_{IT}, N)^{\sigma-1}\right)^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\Gamma(N_{IT}, N)\,\Pi(N_{IT}, N)^{\sigma-1}\right)^{\frac{1}{\sigma}}(A_L L_L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

and using the definitions, $B_1(N_{IT}) = \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz = (1-\Gamma(N_{IT}, N))\Pi(N_{IT}, N)^{\sigma-1}$, $\frac{dB_1}{dN_{IT}} = \gamma_{IT}(N_{IT})^{\sigma-1}$, and $B_2(N_{IT}) = \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$, $\frac{dB_2}{dN_{IT}} =$

$-\gamma_L(N_{IT})^{\sigma-1}$ we rewrite H as

$$\begin{aligned} H &= \left[\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} + \left[\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \right]^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[(B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} + [B_2(N_{IT})]^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Taking the derivative with respect to A_{IT} gives

$$\begin{aligned} \frac{dH}{dA_{IT}} &= \frac{\sigma}{\sigma-1} \left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +(B_2(N_{IT}))^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right]^{\frac{\sigma}{\sigma-1}-1} \frac{1}{\sigma} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} (B_1(N_{IT}))^{\frac{1}{\sigma}-1} \frac{dB_1}{dN_{IT}} \frac{dN_{IT}}{dA_{IT}} \\ &+ \frac{\sigma}{\sigma-1} \left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +(B_2(N_{IT}))^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right]^{\frac{\sigma}{\sigma-1}-1} (B_1(N_{IT}))^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}-1} L_{IT}\dot{A}_{IT} \\ &+ \frac{\sigma}{\sigma-1} \left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +(B_2(N_{IT}))^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right]^{\frac{\sigma}{\sigma-1}-1} \frac{1}{\sigma} (A_L L_L)^{\frac{\sigma-1}{\sigma}} (B_2(N_{IT}))^{\frac{1}{\sigma}-1} \frac{dB_2}{dN_{IT}} \frac{dN_{IT}}{dA_{IT}} \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +[B_2(N_{IT})]^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right]^{\frac{\sigma}{\sigma-1}-1} (\sigma-1)^{-1} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} (B_1(N_{IT}))^{\frac{1}{\sigma}-1} \frac{dB_1}{dN_{IT}} \frac{dN_{IT}}{dA_{IT}} \\ &+ \left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +[B_2(N_{IT})]^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right]^{\frac{\sigma}{\sigma-1}-1} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}-1} L_{IT} \\ &+ \left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +[B_2(N_{IT})]^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right]^{\frac{\sigma}{\sigma-1}-1} (\sigma-1)^{-1} (A_L L_L)^{\frac{\sigma-1}{\sigma}} (B_2(N_{IT}))^{\frac{1}{\sigma}-1} \frac{dB_2}{dN_{IT}} \frac{dN_{IT}}{dA_{IT}} \end{aligned}$$

$$= \frac{H \left[\begin{array}{c} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} (B_1(N_{IT}))^{\frac{1}{\sigma}-1} \frac{dB_1}{dN_{IT}} \frac{dN_{IT}}{dA_{IT}} \\ +(\sigma-1)(B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}-1} L_{IT} \\ +(A_L L_L)^{\frac{\sigma-1}{\sigma}} (B_2(N_{IT}))^{\frac{1}{\sigma}-1} \frac{dB_2}{dN_{IT}} \frac{dN_{IT}}{dA_{IT}} \end{array} \right]}{\left[\begin{array}{c} (B_1(N_{IT}))^{\frac{1}{\sigma}} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}} \\ +[B_2(N_{IT})]^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \end{array} \right] (\sigma-1)}$$

$$
\begin{aligned}
&= H\frac{\begin{pmatrix} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}(B_1(N_{IT}))^{\frac{1}{\sigma}-1}\frac{dB_1}{dN_{IT}} \\ +(A_LL_L)^{\frac{\sigma-1}{\sigma}}(B_2(N_{IT}))^{\frac{1}{\sigma}-1}\frac{dB_2}{dN_{IT}} \end{pmatrix}\frac{dN_{IT}}{dA_{IT}}+(\sigma-1)(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}-1}L_{IT}}{(\sigma-1)\left[(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}+[B_2(N_{IT})]^{\frac{1}{\sigma}}(A_LL_L)^{\frac{\sigma-1}{\sigma}}\right]} \\
&= H\frac{\begin{matrix}\begin{pmatrix} (A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}(B_1(N_{IT}))^{\frac{1}{\sigma}-1}\gamma_{IT}(N_{IT})^{\sigma-1} \\ +(A_LL_L)^{\frac{\sigma-1}{\sigma}}(B_2(N_{IT}))^{\frac{1}{\sigma}-1}(-)\gamma_L(N_{IT})^{\sigma-1} \end{pmatrix}\frac{dN_{IT}}{dA_{IT}} \\ +(\sigma-1)(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}-1}L_{IT}\end{matrix}}{(\sigma-1)\left[(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}+[B_2(N_{IT})]^{\frac{1}{\sigma}}(A_LL_L)^{\frac{\sigma-1}{\sigma}}\right]} \\
&= \frac{\frac{H}{A_{IT}}\left[\begin{matrix}\left(1-\frac{(A_LL_L)^{\frac{\sigma-1}{\sigma}}(B_2(N_{IT}))^{\frac{1}{\sigma}-1}\gamma_L(N_{IT})^{\sigma-1}}{(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}(B_1(N_{IT}))^{\frac{1}{\sigma}-1}\gamma_{IT}(N_{IT})^{\sigma-1}}\right) \\ \frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}}N_{IT}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}(B_1(N_{IT}))^{\frac{1}{\sigma}-1}\gamma_{IT}(N_{IT})^{\sigma-1} \\ +(\sigma-1)(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}\end{matrix}\right]}{(\sigma-1)\left[(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}+[B_2(N_{IT})]^{\frac{1}{\sigma}}(A_LL_L)^{\frac{\sigma-1}{\sigma}}\right]} \\
&= \frac{\frac{H}{A_{IT}}\left[\begin{matrix}\left(1-\left(\frac{(A_LL_L)}{(A_{IT}L_{IT})}\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})}\right)^{\sigma-1}\right) \\ \frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}}N_{IT}(B_1(N_{IT}))^{-1}\gamma_{IT}(N_{IT})^{\sigma-1} \\ +(\sigma-1)\end{matrix}\right](A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}(B_1(N_{IT}))^{\frac{1}{\sigma}}}{(\sigma-1)\left[(B_1(N_{IT}))^{\frac{1}{\sigma}}(A_{IT}L_{IT})^{\frac{\sigma-1}{\sigma}}+[B_2(N_{IT})]^{\frac{1}{\sigma}}(A_LL_L)^{\frac{\sigma-1}{\sigma}}\right]}
\end{aligned}
$$

Using the switching condition from automated tasks to labor tasks (86), $\frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})}=\left(\frac{A_{IT}L_{IT}}{A_LL_L}\right)^{\frac{1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}}$ we obtain

$$
\begin{aligned}
0 &= \frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})}-\left(\frac{A_{IT}L_{IT}}{A_LL_L}\right)^{\frac{1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}} \\
\frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})} &= \left(\frac{A_{IT}L_{IT}}{A_LL_L}\right)^{\frac{1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}} \\
0 &= 1-\left(\frac{(A_LL_L)}{(A_{IT}L_{IT})}\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{A_{IT}L_{IT}}{A_LL_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{\sigma-1}{\sigma}}
\end{aligned}
$$

$1-\left(\frac{(A_LL_L)}{(A_{IT}L_{IT})}\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\gamma_L(N_{IT})}{\gamma_{IT}(N_{IT})}\right)^{\sigma-1}=0$, and thus we can write

$$= H \frac{(\sigma-1)\left(A_{IT}L_{IT}\right)^{\frac{\sigma-1}{\sigma}}\left(B_1\left(N_{IT}\right)\right)^{\frac{1}{\sigma}}}{(\sigma-1)\left[\left(B_1\left(N_{IT}\right)\right)^{\frac{1}{\sigma}}\left(A_{IT}L_{IT}\right)^{\frac{\sigma-1}{\sigma}}+\left[B_2\left(N_{IT}\right)\right]^{\frac{1}{\sigma}}\left(A_L L_L\right)^{\frac{\sigma-1}{\sigma}}\right]} \frac{1}{A_{IT}}$$

$$1 > \eta_{H,A_{IT}} = \left[1 + \frac{\left[B_2\left(N_{IT}\right)\right]^{\frac{1}{\sigma}}\left(A_L L_L\right)^{\frac{\sigma-1}{\sigma}}}{\left[B_1\left(N_{IT}\right)\right]^{\frac{1}{\sigma}}\left(A_{IT}L_{IT}\right)^{\frac{\sigma-1}{\sigma}}}\right]^{-1} > 0$$

Effects on the income share of labor in the service sector and total economy: $\phi_L, \frac{w_L L_L}{Y(t)}$

Effects on labor share of income in service sector: ϕ_L Departing from (24) with $\phi_L = \left(1 + \left(\frac{(1-\Gamma(N_{IT},N))}{\Gamma(N_{IT},N)}\right)^{\frac{1}{\sigma}}\left(\frac{A_{IT}}{A_L}\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}$ and using the definitions, $B_1\left(N_{IT}\right) = \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz = (1-\Gamma(N_{IT},N))\Pi(N_{IT},N)^{\sigma-1}$, $\frac{dB_1}{dN_{IT}} = \gamma_{IT}(N_{IT})^{\sigma-1}$, and $B_2\left(N_{IT}\right) = \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz = \Gamma(N_{IT},N)\Pi(N_{IT},N)^{\sigma-1}$, $\frac{dB_2}{dN_{IT}} = -\gamma_L(N_{IT})^{\sigma-1}$ we rewrite ϕ_L as

$$\phi_L = \frac{1}{\left(\frac{L_{IT}}{L_L}\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}}+1}.$$

Taking the derivative gives:

$$\frac{d\phi_L}{dA_{IT}} = -\frac{1}{(\ldots)^2}\left(\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\left[\begin{array}{c} \frac{\sigma-1}{\sigma}\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}}\left(\frac{A_{IT}}{A_L}\right)^{-\frac{\sigma-1}{\sigma}-1}\frac{1}{A_L} \\ +\frac{1}{\sigma}\left(\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}-1}\left(\frac{\frac{dB1}{dN_{IT}}\frac{dN_{IT}}{dA_{IT}}}{B_2(N_{IT})} - \frac{B_1(N_{IT})\frac{dB_2}{dN_{IT}}\frac{dN_{IT}}{dA_{IT}}}{B_2(N_{IT})^2}\right) \end{array}\right]$$

$$\frac{d\phi_L}{dA_{IT}}\frac{1}{\phi_L} = -\frac{\left(\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}}{(\ldots)\,\sigma}\left[\begin{array}{c} (\sigma-1)\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}}\left(\frac{A_{IT}}{A_L}\right)^{-\frac{\sigma-1}{\sigma}-1}\frac{1}{A_L} \\ +\left(\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}-1}\frac{B_1(N_{IT})}{B_2(N_{IT})}\left(\frac{dB_1}{dN_{IT}}\frac{1}{B_1(N_{IT})}\frac{dN_{IT}}{dA_{IT}} - \frac{dB_2}{dN_{IT}}\frac{1}{B_2(N_{IT})}\frac{dN_{IT}}{dA_{IT}}\right) \end{array}\right]$$

$$\frac{d\phi_L}{dA_{IT}}\frac{1}{\phi_L} = \frac{-\left(\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}}\left(\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}}{(\ldots)\,\sigma}\left[(\sigma-1)\frac{1}{A_{IT}} + \left(\begin{array}{c} \frac{dB_1}{dN_{IT}}\frac{N_{IT}}{B_1(N_{IT})}\frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}} \\ -\frac{dB_2}{dN_{IT}}\frac{N_{IT}}{B_2(N_{IT})}\frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}} \end{array}\right)\frac{1}{A_{IT}}\right]$$

$$\frac{d\phi_L}{dA_{IT}}\frac{1}{\phi_L} = \frac{-\left(\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_1(N_{IT})}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}}\left(\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\left(\frac{L_{IT}}{L_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{A_{IT}}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_1(N_{IT})z}{B_2(N_{IT})}\right)^{\frac{1}{\sigma}}+1\right)\sigma}\left[(\sigma-1) + \left(\begin{array}{c} \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})} \\ +\frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})} \end{array}\right)N_{IT}\eta_{N_{IT},A_{IT}}\right]\frac{1}{A_{IT}}$$

$$\eta_{\phi_L,A_{IT}} = \frac{d\phi_L}{dA_{IT}}\frac{A_{IT}}{\phi_L} = \frac{-1}{\sigma}\frac{(\sigma-1)+\left(\frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})}+\frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})}\right)N_{IT}\eta_{N_{IT},A_{IT}}}{1+\left(\frac{A_L L_L}{A_{IT}L_{IT}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{B_2(N_{IT})}{B_1(N_{IT})}\right)^{\frac{1}{\sigma}}} < 0.$$

For $1 < \sigma$ the share will clearly decline, $\eta_{\phi_L,A_{IT}}$ ¡0. If $1 > \sigma$ the share will not necessarily increase. Introducing more IT tasks $-\left(\frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})}+\frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})}\right)N_{IT}\eta_{N_{IT},A_{IT}} < 0$ will decrease the share of labor income and overcompensate the potentially positive effect from complementarity, $1 > \sigma$.

Effects on labor share of income in the total economy: $\frac{w_L L_L}{Y}$.

$$\frac{w_L L_L}{Y} = \frac{\phi_L}{(1+\alpha)}, \quad \frac{\frac{dw_L L_L/Y}{dA_{IT}}}{w_L L_L/Y} = \frac{d\phi_L}{\phi_L} = \eta_{\phi_L,A_{IT}}\frac{dA_{IT}}{A_{IT}} < 0$$

Effect on income share of technology providers $\frac{w_{it}l_{it}}{Y}$

$$\begin{aligned}
\frac{w_{IT}(t)L_{IT}}{Y(t)} &= \frac{1-\phi_L}{1+\alpha} \\
\frac{d\frac{w_{IT}(t)L_{IT}}{Y(t)}}{\frac{w_{IT}(t)L_{IT}}{Y(t)}} &= \frac{-1}{1+\alpha}\frac{1+\alpha}{1-\phi_L}\frac{\partial\phi_L}{\partial A_{IT}}\frac{dA_{IT}}{A_{IT}}A_{IT} \\
&= \frac{-1}{\frac{1}{\phi_L}-1}\frac{\partial\phi_L}{\partial A_{IT}}\frac{A_{IT}}{\phi_L}\frac{dA_{IT}}{A_{IT}}
\end{aligned}$$

If A_{IT} is repeatedly increasing, the limit is:

$$\lim_{A_{IT}\to\infty}\frac{w_{IT}(t)L_{IT}}{Y(t)} = \frac{1-\phi_L}{1+\alpha} = \frac{1}{1+\alpha} - 0.$$

Effects on consumption rate: c

$$\begin{aligned}
c &= \frac{\phi_L}{(1+\alpha)}, \quad dc = \frac{\phi_L}{(1+\alpha)}\frac{\partial\phi_L}{\partial A_{IT}}\frac{dA_{IT}}{A_{IT}}\frac{A_{IT}}{\phi_L} \\
\frac{dc}{c}\frac{A_{IT}}{dA_{IT}} &= \frac{\partial\phi_L}{\partial A_{IT}}\frac{A_{IT}}{\phi_L} = \eta_{c,A_{IT}} = \eta_{\phi_L,A_{IT}}
\end{aligned}$$

Effects of human service in production: H_Q We reconsider the discussion of the implicit function $\tilde{H}_Q$ and restate the effect of a change of A_{IT} on H_Q as we have derived in section F of this Appendix.

$$\begin{aligned}
\frac{dH_Q}{dA_{IT}} &= -\frac{\frac{\partial F}{\partial A_{IT}}}{\frac{\partial F}{\partial \tilde{H}_Q}} = -\frac{-c\left[\overset{<0}{\eta_{c,A_{IT}}} H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi}}{\left[(2-\varphi)\, cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right] H_Q^{-\varphi}} \\
\eta_{H_Q,A_{IT}} &= \frac{dH_Q}{H_Q}\frac{A_{IT}}{dA_{IT}} = \frac{\frac{\phi_L}{(1+\alpha)}\left[\overset{<0}{\eta_{\phi_L,A_{IT}}} H_\phi + \overset{>0}{\eta_{H,A_{IT}}} H\right]}{\left[(2-\varphi)\, cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]} > 0
\end{aligned}$$

Effects on deployment rate: ω

$$\begin{aligned}
\omega\left(t\right) &= \frac{\tilde{Y}(t)}{\tilde{Y}^P(t)} = \frac{\tilde{H}_Q(t)}{H} \\
d\omega &= \left[\eta_{H_Q,A_{IT}} - \eta_{H,A_{IT}}\right]\frac{dA_{IT}}{A_{IT}} = \left(\frac{d\tilde{H}_Q(t)}{dA_{IT}}\frac{A_{IT}}{\tilde{H}_Q} - \frac{dH}{dA_{IT}}\frac{A_{IT}}{H}\right)\frac{\tilde{H}_Q}{H}\frac{dA_{IT}}{A_{IT}} =
\end{aligned}$$

$$\frac{d\omega}{\omega} = \left[\frac{c\left(\eta_{\phi_L,A_{IT}} H_\phi + H\eta_{H,A_{IT}}\right)}{(2-\varphi)\, cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)} - \eta_{H,A_{IT}}\right]\frac{dA_{IT}}{A_{IT}}$$

In order to have a negative deployment effect we want to show that

$$\frac{\frac{\phi_L}{(1+\alpha)}\left[\eta_{\phi_L,A_{IT}} H_\phi + H\eta_{H,A_{IT}}\right]}{\left[(2-\varphi)\, cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]} - \eta_{H,A_{IT}} < 0$$

$$\begin{aligned}
c\left[\eta_{\phi_L,A_{IT}} H_\phi + H\eta_{H,A_{IT}}\right] &< \eta_{H,A_{IT}}\left[(2-\varphi)\, cH_Q - (1-\varphi)\, cov\left(\lambda,\varepsilon_F\right) - (1-\varphi)\, cH\right] \\
c\frac{\eta_{\phi_L,A_{IT}}}{\eta_{H,A_{IT}}} H_\phi + cH &< (2-\varphi)\, cH_Q - (1-\varphi)\, cov\left(\lambda,\varepsilon_F\right) - (1-\varphi)\, cH \\
c\frac{\eta_{\phi_L,A_{IT}}}{\eta_{H,A_{IT}}} H_\phi &< (2-\varphi)\, cH_Q - (1-\varphi)\, cov\left(\lambda,\varepsilon_F\right) - 2cH + \varphi cH \\
c\frac{\eta_{\phi_L,A_{IT}}}{\eta_{H,A_{IT}}} H_\phi &< (2-\varphi)\, cH_Q - (2-\varphi)\, cH - (1-\varphi)\, cov\left(\lambda,\varepsilon_F\right) \\
c\frac{\overset{<0}{\eta}_{\phi_L,A_{IT}}}{\eta_{H,A_{IT}}} H_\phi &< -\left(2-\varphi\right) c\left[H - H_Q\right] - (1-\varphi)\, cov\left(\lambda,\varepsilon_F\right)
\end{aligned}$$

A sufficient condition for this suggestion is

$$\begin{aligned} 0 &< -(2-\varphi)\, c\,[H-H_Q] - (1-\varphi)\, cov\,(\lambda, \varepsilon_F) \\ \frac{(2-\varphi)}{(1-\varphi)} cH_\phi &< -cov\,(\lambda, \varepsilon_F) \end{aligned}$$

and this condition holds if $cov\,(\lambda, \varepsilon_F)$ is sufficiently negative, and thus, for a large variety of parameters.

Effects on wages: $w_L\,(t)$ Due to space limitations, detailed calculations are available from the authors on request.

$$\frac{dw_L(t)}{w_L(t)} = \left[\overset{(ii)}{} \overset{<0}{\eta_{c,A_{IT}}} + \overset{>0}{\eta_{H_Q,H}} \overset{(i)}{} \overset{>0}{\eta_{H,A_{IT}}} \right] \frac{dA_{IT}}{A_{IT}} \lesseqgtr 0$$

$$\begin{aligned} w_L &= \phi_L \frac{p_H H}{L_L}, \quad p_H\,(t) = (1-\alpha) \frac{Q\,(t)}{H} \\ w_L &= \phi_L (1-\alpha) \frac{Q}{H} \frac{H}{L_L} = \phi_L (1-\alpha) \frac{Q}{L_L} \\ \frac{dw_L}{dA_{IT}} &= \frac{d\phi_L}{dA_{IT}} (1-\alpha) \frac{Q}{L_L} + \phi_L (1-\alpha) \frac{1}{L_L} \frac{dQ}{dA_{IT}} \\ \frac{dQ}{dA_{IT}} &= N_i\,(t)\, H_{Qi}^{1-\alpha} x_i^\alpha\,(t) \frac{dH_Q}{dA_{IT}} \frac{1}{H_Q} = Q \frac{dH_Q}{dA_{IT}} \frac{1}{H_Q} \end{aligned}$$

$$\begin{aligned} \frac{dw_L}{dA_{IT}} &= \frac{d\phi_L}{dA_{IT}} \frac{A_{IT}}{\phi_L} \phi_L (1-\alpha) \frac{Q}{L_L} + \phi_L (1-\alpha) \frac{Q}{L_L} \frac{dH_Q}{dA_{IT}} \frac{A_{IT}}{H_Q} \\ \frac{dw_L}{dA_{IT}} \frac{A_{IT}}{w_L} &= \frac{d\phi_L}{dA_{IT}} \frac{A_{IT}}{\phi_L} + \frac{dH_Q}{dA_{IT}} \frac{A_{IT}}{H_Q} = \overset{<0}{\eta}_{\phi_L,A_{IT}} + \overset{>0}{\eta}_{H_Q,A_{IT}} \end{aligned}$$

$$\begin{aligned} \eta_{\phi_L,A_{IT}} &= \frac{d\phi_L}{dA_{IT}} \frac{A_{IT}}{\phi_L} = \frac{1}{\sigma} \frac{-(\sigma-1) - \left(\frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{B_1(N_{IT})} + \frac{\gamma_L(N_{IT})^{\sigma-1}}{B_2(N_{IT})} \right) N_{IT} \eta_{N_{IT},A_{IT}}}{1 + \left(\frac{A_L L_L}{A_{IT} L_{IT}} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{B_2(N_{IT})}{B_1(N_{IT})} \right)^{\frac{1}{\sigma}}} < 0 \\ \eta_{H_Q,A_{IT}} &= \frac{dH_Q}{H_Q} \frac{A_{IT}}{dA_{IT}} = \frac{\frac{\phi_L}{(1+\alpha)} \left[\overset{<0}{\eta_{\phi_L,A_{IT}}} H_\phi + \overset{>0}{\eta_{H,A_{IT}}} H \right]}{[(2-\varphi)\, cH_Q + (1-\varphi)\,(-cov\,(\lambda, \varepsilon_F) - cH)]} > 0 \end{aligned}$$

$$
\begin{aligned}
\eta_{w_L,A_{IT}} &= \eta_{\phi_L,A_{IT}} + \frac{\frac{\phi_L}{(1+\alpha)}\left[\overset{<0}{\eta_{\phi_L,A_{IT}}}H_\phi + \overset{>0}{\eta_{H,A_{IT}}}H\right]}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)} < 0 \\
&= \frac{\eta_{\phi_L,A_{IT}}\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)\right] + c\left[\eta_{\phi_L,A_{IT}}H_\phi + \eta_{H,A_{IT}}H\right]}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)} \\
&= \frac{\eta_{\phi_L,A_{IT}}\left(2-\varphi\right)cH_Q + c\eta_{\phi_L,A_{IT}}H_\phi + \eta_{\phi_L,A_{IT}}\left(1-\varphi\right)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right) + c\eta_{H,A_{IT}}H}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\eta_{\phi_L,A_{IT}}\left(1-\varphi\right)cH_Q + \eta_{\phi_L,A_{IT}}cH + \eta_{\phi_L,A_{IT}}\left(1-\varphi\right)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right) + c\eta_{H,A_{IT}}H}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)} \\
&= \frac{\eta_{\phi_L,A_{IT}}\left(1-\varphi\right)cH_Q + \eta_{\phi_L,A_{IT}}cH - cov\left(\lambda,\varepsilon_F\right)\eta_{\phi_L,A_{IT}}\left(1-\varphi\right) - \eta_{\phi_L,A_{IT}}cH\left(1-\varphi\right) + c\eta_{H,A_{IT}}H}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)} \\
&= \frac{\left[\begin{array}{c} \eta_{\phi_L,A_{IT}}\left(1-\varphi\right)cH_Q + \eta_{\phi_L,A_{IT}}cH - cov\left(\lambda,\varepsilon_F\right)\eta_{\phi_L,A_{IT}}\left(1-\varphi\right) \\ -\eta_{\phi_L,A_{IT}}cH + \eta_{\phi_L,A_{IT}}c\varphi H + c\eta_{H,A_{IT}}H \end{array}\right]}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)} \\
&= \frac{\eta_{\phi_L,A_{IT}}\left(1-\varphi\right)cH_Q - cov\left(\lambda,\varepsilon_F\right)\eta_{\phi_L,A_{IT}}\left(1-\varphi\right) + \eta_{\phi_L,A_{IT}}c\varphi H + c\eta_{H,A_{IT}}H}{(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right)-cH\right)} < 0
\end{aligned}
$$

$$
0 < \eta_{\phi_L,A_{IT}}\left(1-\varphi\right)cH_Q - cov\left(\lambda,\varepsilon_F\right)\eta_{\phi_L,A_{IT}}\left(1-\varphi\right) + \eta_{\phi_L,A_{IT}}c\varphi H + c\eta_{H,A_{IT}}H < 0
$$

$$
\begin{aligned}
-c\frac{\eta_{H,A_{IT}}}{\eta_{\phi_L,A_{IT}}}H &< \left(1-\varphi\right)cH_Q - cov\left(\lambda,\varepsilon_F\right)\left(1-\varphi\right) + c\varphi H \\
-cov\left(\lambda,\varepsilon_F\right)\left(1-\varphi\right) &> -c\left(\left(\frac{\eta_{H,A_{IT}}}{\eta_{\phi_L,A_{IT}}}-\varphi\right)H - \left(1-\varphi\right)H_Q\right) \\
cov\left(\lambda,\varepsilon_F\right) &< c\left(\left(\frac{\eta_{H,A_{IT}}}{\eta_{\phi_L,A_{IT}}}-\varphi\right)\frac{H}{(1-\varphi)} - H_Q\right)
\end{aligned}
$$

Effects on the growth rate $\frac{d\tilde{g}_Y}{\tilde{g}_Y}$: From Equation (52) we know

$$
g_N = \left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q\right)^{\varphi} \left(g_A\right)^{1-\varphi}.
$$

Taking the derivative gives

$$
\begin{aligned}
\frac{dg_N}{dA_{IT}} &= \varphi\left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} H_Q\right)^{\varphi-1} \left(g_A\right)^{1-\varphi} \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} \frac{dH_Q}{dA_{IT}} \\
\eta_{g_N,A_{IT}} &= \frac{dg_N}{dA_{IT}}\frac{A_{IT}}{g_N} = \varphi\eta_{H_Q,A_{IT}} = \varphi\frac{dH_Q}{dA_{IT}}\frac{A_{IT}}{H_Q} > 0
\end{aligned}
$$

I. Simultaneous demand shocks

Simultaneous demand shock: We depart from equation (52)

$$\frac{dg_N}{dA_{IT}} = \varphi g_N \frac{1}{H_Q} \frac{dH_Q^{\text{total}}}{dA_{IT}}.$$

In addition to the pure supply side mechanism which we derived in (78) we now consider a simultaneous demand shock with $\frac{dv}{dA_{IT}} < 0$. Thus, labor in production is now determined by

$$\begin{aligned}
\frac{dH_Q^{\text{total}}}{dA_{IT}} \frac{A_{IT}}{H_Q} &= \frac{\partial H_Q^{\text{supply}}}{\partial A_{IT}} \frac{A_{IT}}{H_Q} + \frac{\partial H_Q^{\text{demand}}}{\partial A_{IT}} \frac{A_{IT}}{H_Q} \\
\eta_{H_Q,A_{IT}}^{\text{total}} &= \eta_{H_Q,A_{IT}}^{\text{supply}} + \eta_{H_Q,A_{IT}}^{\text{demand}}
\end{aligned}$$

While $\frac{\partial H_Q^{\text{supply}}}{\partial A_{IT}} > 0$ is the effect derived in (78) we need to determine $\frac{\partial H_Q^{\text{demand}}}{\partial A_{IT}} = \frac{dH_Q}{dv} \frac{dv}{dA_{IT}}$. With $\frac{dH_Q}{dv} = -\frac{\frac{cov(\lambda,\varepsilon_F)}{(1-\alpha^2)} \left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}} (g_A)^{1-\varphi}}{\left[(2-\varphi)cH_Q+(1-\varphi)(-cov(\lambda,\varepsilon_F)-cH)\right] H_Q^{-\varphi}} > 0$ (see appendix F and (62))

$$\begin{aligned}
\frac{\partial H_Q^{\text{supply}}}{\partial A_{IT}} &= -\frac{-c\left[\overset{<0}{\eta_{c,A_{IT}}} H_\phi + H \overset{>0}{\eta_{H,A_{IT}}}\right] \frac{1}{A_{IT}} H_Q^{1-\varphi}}{\left[(2-\varphi) cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right] H_Q^{-\varphi}} \\
\eta_{H_Q,A_{IT}}^{\text{supply}} &= \frac{\partial H_Q^{\text{supply}}}{\partial A_{IT}} \frac{A_{IT}}{H_Q} = \frac{c\left[\overset{<0}{\eta_{c,A_{IT}}} H_\phi + H \overset{>0}{\eta_{H,A_{IT}}}\right]}{\left[(2-\varphi) cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]} \\
\frac{\partial H_Q^{\text{demand}}}{\partial A_{IT}} &= -\frac{\frac{cov(\lambda,\varepsilon_F)}{(1-\alpha^2)} \left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}} (g_A)^{1-\varphi}}{\left[(2-\varphi) cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right] H_Q^{-\varphi}} \frac{dv}{dA_{IT}} < 0 \\
\eta_{H_Q,A_{IT}}^{\text{demand}} &= \frac{\partial H_Q^{\text{demand}}}{\partial A_{IT}} \frac{A_{IT}}{H_Q} = -\frac{\frac{cov(\lambda,\varepsilon_F)}{(1-\alpha^2)} v \left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}} (g_A)^{1-\varphi}}{\left[(2-\varphi) cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right] H_Q^{1-\varphi}} \frac{A_{IT}}{v} \frac{dv}{dA_{IT}} < 0.
\end{aligned}$$

Combining both elements gives

$$
\begin{aligned}
\frac{dH_Q^{\text{total}}}{dA_{IT}} &= \frac{c\left[\overbrace{\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}}^{>0}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi} - \frac{cov(\lambda,\varepsilon_F)}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{1-\varphi}\frac{dv}{dA_{IT}}}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]H_Q^{-\varphi}} < 0 \\
0 &> c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi} - \frac{cov\left(\lambda,\varepsilon_F\right)}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{1-\varphi}\frac{dv}{dA_{IT}}
\end{aligned}
$$

$$
\begin{aligned}
c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi} &< \frac{cov\left(\lambda,\varepsilon_F\right)}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{1-\varphi}\frac{dv}{dA_{IT}} \\
\frac{c\left(1-\alpha^2\right)\overbrace{\left[\eta_{c,A_{IT}}H_\phi + \eta_{H,A_{IT}}H\right]}^{>0}}{v\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}}\frac{\left(\frac{H_Q}{g_A}\right)^{1-\varphi}}{\underset{<0}{\eta_{v,A_{IT}}}} &> \overset{<0}{cov\left(\lambda,\varepsilon_F\right)}
\end{aligned}
$$

This holds if in absolute terms $cov\left(\lambda,\varepsilon_F\right)$ is sufficiently large.

$$
\eta_{g_N,A_{IT}} = \frac{dg_N}{dA_{IT}}\frac{A_{IT}}{g_N} = \varphi\left[\frac{c\left[\eta_{c,A_{IT}}H_\phi + \eta_{H,A_{IT}}H\right]}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]}\right] - \frac{\frac{cov(\lambda,\varepsilon_F)}{(1-\alpha^2)}v\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(\frac{g_A}{H_Q}\right)^{1-\varphi}}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\right.\right.\right.}
$$

Simultaneous innovation shock: The effect on the growth rate

$$
\begin{aligned}
\frac{dg_N}{dA_{IT}} &= \varphi\left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}}H_Q\right)^{\varphi-1}(g_A)^{1-\varphi}\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}}\frac{dH_Q}{dA_{IT}} + (1-\varphi)\left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}}H_Q\right)^{\varphi}(g_A)^{-\varphi}\frac{dg_A}{dA_{IT}} \\
&= \varphi g_N\frac{1}{H_Q}\frac{dH_Q}{dA_{IT}} + (1-\varphi)\,g_N\frac{1}{g_A}\frac{dg_A}{dA_{IT}} \\
\eta_{g_N,A_{IT}} &= \frac{dg_N}{dA_{IT}}\frac{A_{IT}}{g_N} = \varphi\eta_{H_Q,A_{IT}} + (1-\varphi)\,\eta_{g_A,A_{IT}}
\end{aligned}
$$

As in the previsous section of this appemdix we depart from

$$
\eta_{H_Q,A_{IT}}^{\text{total}} = \eta_{H_Q,A_{IT}}^{\text{supply}} + \eta_{H_Q,A_{IT}}^{\text{demand}}.
$$

While the supply effect $\frac{\partial H_Q^{\text{supply}}}{\partial A_{IT}} > 0$ is known [see (78)] we need to determine $\frac{\partial H_Q^{\text{demand}}}{\partial A_{IT}} =$

$\frac{dH_Q}{dg_A}\frac{dg_A}{dA_{IT}}$, the effect related to the demand side. And we follow the idea that an increase in AI leads to a shift in resoources that simultaneously reduces the product innovations rate g_A, with $\frac{dg_A}{dA_{IT}} < 0$. Thus, labor in production is now determined (see appendix F and (62)) by

$$
\begin{aligned}
\frac{\partial H_Q^{\text{supply}}}{\partial A_{IT}} &= \frac{c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi}}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]H_Q^{-\varphi}} \\
\eta_{H_Q,A_{IT}}^{\text{supply}} &= \frac{c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]} \\
\frac{\partial H_Q^{\text{demand}}}{\partial A_{IT}} &= -\frac{\frac{(1-\varphi)cov(\lambda,\varepsilon_F)\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{-\varphi}}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]H_Q^{-\varphi}}\frac{dg_A}{dA_{IT}} > 0 \\
\eta_{H_Q,A_{IT}}^{\text{demand}} &= \frac{-\frac{(1-\varphi)cov(\lambda,\varepsilon_F)\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}\left(\frac{g_A}{H_Q}\right)^{1-\varphi}}{\left[(2-\varphi)\,cH_Q + (1-\varphi)\left(-cov\left(\lambda,\varepsilon_F\right) - cH\right)\right]}\frac{A_{IT}}{g_A}\frac{dg_A}{dA_{IT}}
\end{aligned}
$$

The overall effect on H_Q is $\eta_{H_Q,A_{IT}}^{\text{total}} = \eta_{H_Q,A_{IT}}^{\text{supply}} + \eta_{H_Q,A_{IT}}^{\text{demand}}$ which is negative if

$$
\frac{dH_Q^{\text{total}}}{dA_{IT}} = c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi} - \frac{(1-\varphi)\,cov\left(\lambda,\varepsilon_F\right)\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{-\varphi}\frac{dg_A}{dA_{IT}} < 0
$$

$$
\begin{aligned}
c\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]\frac{1}{A_{IT}}H_Q^{1-\varphi} &< \frac{(1-\varphi)\,cov\left(\lambda,\varepsilon_F\right)\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}(g_A)^{-\varphi}\frac{dg_A}{dA_{IT}} \\
\frac{c\left(1-\alpha^2\right)}{v\left(1-\varphi\right)}\frac{\left[\overset{<0}{\eta_{c,A_{IT}}}H_\phi + H\overset{>0}{\eta_{H,A_{IT}}}\right]}{\left(\frac{\alpha^2}{c_x}\right)^{\frac{\varphi-\alpha}{1-\alpha}}}\frac{\left(\frac{H_Q}{g_A}\right)^{1-\varphi}}{\underset{<0}{\eta_{g_A,A_{IT}}}} &> cov\left(\lambda,\varepsilon_F\right)
\end{aligned}
$$

and thus,

$$
\eta_{H_Q,A_{IT}}^{\text{total}} < 0
$$

With this effect on H_Q and the assumption $\frac{dg_A}{dA_{IT}} < 0$ we can now determine the overall effect on the growth rate [from $g_N = \left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}}H_Q\right)^{\varphi}(g_A)^{1-\varphi}$] and obtain

$$
\eta_{g_N,A_{IT}} = \varphi\overset{<0}{\eta_{H_Q,A_{IT}}} + (1-\varphi)\overset{<0}{\eta_{g_A,A_{IT}}} < 0
$$

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