New Developments in Models of Search in the Labor Market

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Abstract: Equilibrium models of labor markets characterized by search and recruiting friction and by the need to reallocate workers from time to time across alternative productive activities represent the segment of the research frontier explored in this paper. In this literature, unemployment spell and job spell durations as well as wage offers are treated as endogenous outcomes of forward looking job creation and job destruction decisions made by the workers and employers who populate the models. The solutions studied are dynamic stochastic equilibria in the sense that time and uncertainty are explicitly modeled, expectations are rational, private gains from trade are exploited, and the actions taken by all agents are mutually consistent. We argue that the framework provides a useful setting in which to study the effects of alternative wage setting institutions and different labor market policy regimes.

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1 Introduction

Equilibrium models of labor markets characterized by search and recruiting friction and by the need to reallocate workers from time to time across alternative productive activities represent the segment of the research frontier explored in this paper. In this literature, unemployment spell and job spell durations as well as wage offers are treated as endogenous outcomes of forward looking job creation and job destruction decisions made by the workers and employers who populate the models. The solutions studied are dynamic stochastic equilibria in the sense that time and uncertainty are explicitly modeled, expectations are rational, private gains from trade are exploited, and the actions taken by all agents are mutually consistent. In contrast to the earlier literature on individual worker job search decisions, for example, much of that reviewed by Mortensen (1986), the equilibrium search approach explicitly accounts for and indeed emphasizes the role of employers on the demand side of the labor market. As a consequence, we argue, the framework provides a rich and useful setting in which to study the effects of alternative wage setting institutions and different labor market policy regimes.

The need for a richer equilibrium framework for labor market analysis then that provided by the frictionless competive model is both empirical and conceptual. Large numbers of workers and jobs flow between inactivity and market production at the aggregate level. At the level of individual workers and employers, worker flows between labor market states and job creation and job destruction flows are reflected in activity spells found in panel data whose durations reflect the time spent searching for work, filling a vacancy, and working in a particular job. These movements are concealed in existing models of employment that focus on stocks. The emphasis on mobility makes the types of models reviewed here part of the so-called flows approach (See Blanchard and Diamond (1992).) to labor market analysis.[1]

Still another empirical reason for interest in the framework is wage dispersion across observably identical workers. These differentials have led many observers to question and some to reject perfectly competitive wage theory. Search and matching frictions inevitably generate match specific rents that the wage must divide between worker and employer. Because the precise way in which these rents might be shared is indeterminate, the framework requires some alternative to the marginal productivity theory of wages, at least in its simplest form. Although the natural and usual specification is *ex post* bargaining in the models reviewed, alternatives such as a monopoly

[1]We have also written a companion paper, Mortensen and Pissarides (1998), on the macroeconomic implications of the flows approach for the forthcoming *Handbook of Macroeconomics*. For that reason, the focus of this paper is more microeconomic.

union specification, an insider-outsider story, and efficiency wage theories can all be accommodated and studied within the framework. The principal alternative to the wage as a bargaining outcome, that studied most extensively in the literature, is the assumption that employers post wage offers.

What new lessons can be learned about the effects of policy using the new approach? Because unemployment has an economic role in the flows framework, welfare statements about the effects of policy on unemployment and on the cost of unemployment experienced by those who bear it are possible. Also, the total effect of a policy can be decomposed into effects on unemployment duration and on unemployment incidence. As a consequence of this fact and the two sided nature of the models, multiple channels of influence arise. For example, unemployment benefits influence both worker incentives to accept employment and the worker credible wage demand. Because the wage affects employer incentives to create vacancies and recruit workers, the total impact on unemployment duration is a consequence of decisions made on both sides of the labor market. As the wage impacts job destruction as well, there are a least three different channels through which unemployment insurance benefits might be expected to affect unemployment.

Consider the effects of different forms of employment subsidies as another example of fruitful policy application. In conventional static models, a hiring subsidy is treated as a reduction in labor costs which increases the demand for labor. Whether the subsidy is paid to employers on a per employee hired basis or is proportional to the employment stock is immaterial. Because these two alternative subsidy forms have different effects on the job creation and job destruction decisions that determine worker and employer flows, questions of the kind recently raised by Phelps (1997) about the form that employment subsidies should take can be analyzed within the equilibrium flows framework.

The comments above suggest that the existing literature on equilibrium search forms a unified whole. Although related, there are two quite different branches of the search equilibrium literature, each with its own primary concerns. The goal of the first is to explain worker and job flows and levels of unemployment within the rational forward looking agent paradigm. Fundamental is the idea that two-sided frictions exist in the process of matching trading partners and that agents on both sides of a market make investments in overcoming them. As a result, the job creation flow depends on the numbers of unemployed workers and vacant jobs available and on the intensities with workers search and employers recruit, a relationship which has become known as the matching function. The effects of market friction on the incentives to investment in search, recruiting, training and other forms of match specific capital which in turn determine the equilibrium level of employment are the primary concerns in the literature based on the "matching approach" to labor market analysis.

Contributors to the second literature show that wage dispersion can be an equilibrium outcome in markets with friction. By assumption, wage offers are set by employers in a non-cooperative setting while workers search for the best among them. Here, search friction is regarded as simply the time required for workers to gather information about wage offers. The outcomes of these strategic "wage posting" games are studied as explanations of wage differentials that are not associated with observed worker skill.

Although a review of recent developments in search equilibrium is a principal purpose of the paper, it is not the exclusive one. Another goal is to show how the general approach has and can be used to study the employment effects of different wage determination mechanisms and can be applied to labor market policy analysis. We also show how the two branches of the search equilibrium literature can be reintegrated and suggest some of the rewards that such a synthesis offers.

The remainder of paper is composed of seven sections. The tools used and the concepts applied by contributors to the literature on search equilibrium market models are briefly introduced in section 2. Formal models of job-worker matching, labor market flows, and equilibrium unemployment are the topics of section 3. In these models, wages are determined by a specific rent sharing rule that can viewed as the outcome of a Nash bargain between worker and employer engaged in when they meet. Section 4 reviews variations of the matching model characterized by different wage determination mechanisms. In section 5, applications of the matching approach to the analysis of labor market policy are reviewed and illustrated. Forms of wage dispersion that arises as equilibrium outcomes of wage posting games are the principal topics of section 6. The implications of a synthesis of the matching and wage posting approaches to modeling labor market equilibrium are sketched in section 7. Finally, a very brief summary concludes the paper.

2 Modeling Markets with Friction

Market friction, the costly delay in the process of finding trading partners and determining the terms of trade, is ignored in the standard theory of perfectly competitive markets. Friction is explicitly modeled in the work reviewed in this paper. The central problem of the theory of markets with friction is to find a useful way to make the behavior of individual agents both individually rational and mutually consistent. In this section, we survey the concepts introduced in the recent literature on the problem.[2]

2.1 The stopping problem

The tools of dynamic optimization applied in the equilibrium search literature are introduced first. We do so in the process of reviewing the sequential job search model, the workhorse of the literature, which is based on the decision theoretic optimal stopping problem.

A distribution of payoffs characterized by a c.d.f. $F(W)$ is postulated which is known to the searcher. A sequential sample of realizations can be drawn with replacement at a constant per observation cost denoted by a. Only one of the realizations can be accepted and acceptance is a sequential decision without recall. A search strategy determines when to accept, i.e., it is a stopping rule. An *optimal stopping strategy* maximizes the expected present value of the realization accepted net of the accumulated costs of search.

Application of the model to the job search problem in which workers are not fully informed about the terms of available employment offers is simply a matter of interpretation of this structure. Think of the sampling process as that of sequentially applying for jobs selected at random and let each realization of W represent the value of an offered employment contract, either the wage or more generally the present value of a worker's future utility stream conditional on accepting the offer. In discrete time, the stopping decision is easily formulated as a dynamic programming problem. If a single sample is taken in every period until the process stops and past realizations cannot be recalled, then the value of searching in each period, U_t, is generated by the Bellman equation

$$U_t = \frac{b-a}{1+r} + \frac{1}{1+r}\int \max\{W, U_{t+1}\} dF(W), \; t = 1, 2.... \quad (1)$$

where r is the discount or risk free interest rate, b is income flow received contingent on unemployment less any cost of searching for a job, and a represents the cost of search per period. Namely, the optimal strategy involves comparing the observed current realization of the sampling process W_t with the value of continued search U_{t+1} in the next period. If the former exceeds the latter, then the search process stops, i.e., the optimal strategy satisfies a *reservation property.* In the infinite horizon case, the value of continued

[2] Some of the original papers that raised the issues discussed in this section include Diamond and Maskin (1979), Pissarides (1979), Diamond (1982), Mortensen (1982), and Pissarides (1984).

search is the stationary solution to equation (1), i.e., $U_t = U$ for all t where

$$U = \frac{b-a}{1+r} + \frac{1}{1+r}\int \max\{W, U\} dF(W), \tag{2}$$

As the right side is a contraction map for all $r > 0$,[3] call it $T(U)$, a unique finite solution $U = T(U)$ exists provided that the c.d.f. F has a first moment.

Virtually all the literature on equilibrium search is cast in continuous rather than discrete time. Although this fact is partially a historical accident, continuous time techniques can often reduce the apparent complexity of sequential search and recruiting problems. For example, allowing for a stochastic time interval between offer arrivals is one realistic extension easier to formalize in continuous time. Because arrival dates are separated in continuous time, decisions are revised only after arrivals. Hence, the analysis reduces to a dynamic programming formulation in which the time intervals between decision dates is a strictly positive random variable with a know duration distribution.

Characterize the distribution of random waiting time between offer arrivals by its generally duration dependent hazard function $\lambda(t)$, i.e., the probability that an offer will not arrive before T, the associated survivor function of the waiting time distribution, is $\exp\{-\int_0^T \lambda(t)dt\}$. Taking account of the waiting duration, the Bellman equation for the extended model becomes

$$\begin{aligned} U(t) &= E_T\left\{(b-a)\int_t^T e^{-r*(s-t)}ds + e^{-r(T-t)}\int \max\{W, U(T)\}dF(W)\right\} \quad (3) \\ &= \int_t^\infty \left((b-a)\int_t^T e^{-r(s-t)}ds + e^{-r(T-t)}\int \max\{W, U(T)\}dF(W)\right) \\ &\quad \times \lambda(T)e^{-\int_0^T \lambda(t)dt}dT \end{aligned}$$

where $U(t)$ is the value of search at time t and $T > t$ is the future random date at which the first offer arrives. Given an exponential waiting time distribution, a constant hazard $\lambda(t) \equiv \lambda$, the value of search is stationary and solves

$$\begin{aligned} U &= \int_0^\infty \left[\frac{b-a}{r}(1-e^{-rT}) + e^{-rT}\int \max\{W, U\}dF(W)\right]\lambda e^{-\lambda T}dT \quad (4) \\ &= \frac{b-a}{r+\lambda} + \frac{\lambda}{r+\lambda}\int \max\{W, U\}dF(W) \end{aligned}$$

[3] It satisfies Blackwell sufficient conditions. (See Lucas and Stokey (1989), p 54.)

where λ is the Poisson offer arrival rate. Note that (4) is a simple generalization of (2) that accounts for the duration of the waiting period, $1/\lambda$ in the exponential case.

The full power of the continuous time formulation is suggested by the following equivalent "asset pricing" representation of equation (4):

$$rU = b - a + \lambda \int [\max\{W, U\} - U] dF(W) \tag{5}$$

U represents the "asset" or "option" value of search activity. Given this interpretation, equation (5) simply prices the option by requiring that the opportunity cost of holding it, the left hand side, is equal to the current income flow, $b - a$, plus the expected capital gain flow, the product of the arrival frequency λ and the expected capital gain given an offer arrival.

In the general case of a non-stationary arrival hazard and cost of search cost, both exhibiting duration dependence, one can show that $U(t)$ must be a solution to the generalized asset pricing equation, the differential equation

$$rU(t) = b(t) - a(t) + \lambda(t) \int [\max\{W, U(t)\} - U(t)] dF(W) + \frac{dU(t)}{dt} \tag{6}$$

where the duration derivative dU/dt is the pure rate of capital gain or loss attributable to waiting another instant for an offer arrival. This equation can be obtained directly by differentiating both side of (3) with respect to t. Using the fact that

$$\frac{dU(t)}{dt} = \lim_{dt \to 0} \left\{ \frac{U(t + dt) - U(t)}{dt} \right\},$$

one can also write

$$U(t) = \frac{1}{1 + rdt} \left[\begin{array}{c} (b(t) - a(t))\, dt \\ +\lambda(t) dt \int \max\{W, U\} dF(W) + [1 - \lambda(t) dt]\, U(t + dt) \end{array} \right]$$

as an approximation to equation (6) for all sufficiently small values of the period length $dt > 0$. Obviously, this relationship has a natural interpretation as a Bellman equation in a discrete time formulation of the problem where $\lambda(t)dt$ is the probability of an offer arrival duration the period $(t, t + dt)$ and rdt is the discount rate for the specified period length dt. Of course, time paths for the value of an optimal search strategy must also satisfy the transversality condition $\lim_{t \to \infty}\{U(t)e^{-rt}\} = 0$. The general fact that the option value of search solves a general asset pricing equation and transversality condition of this form provides a very quick and powerful characterization of optimality conditions in equilibrium search models in continuous time.[4]

Interest among empirical labor economists in search theory was generated initially by the fact that it addressed observations on unemployment spell duration lengths and subsequent accepted wage distribution in panel data. As the unemployment spell hazard is $\lambda(t)[1 - F(U(t))]$ and the conditional acceptable wage distribution conditional on duration is $F(W)/[1-F(U(t))]$, the model can be formally applied to interpret available data and to generate testable empirical hypotheses. There is now a substantial literature that does just that.[5] A principal shortcoming of most of it, however, is that only generalizations of this decision theoretic formulation of the optimal stopping problem are typically applied. General equilibrium considerations that may well make primitives endogenous, the offer arrival rate and the distribution offers in particular, are not raised in much of the literature. The remainder of this review points out these equilibrium effects and reviews the existing empirical work that takes them into account.

2.2 Two-sided search and wage determination

In labor, marriage, and related markets, the central problem is the creation of cooperating coalitions composed of two or more agents of different types, e.g., worker and employer, men and women, etc. In the labor market case, a cooperating coalition is a producing unit composed of a job-worker match. The job-worker match is formed when a qualified unemployed worker and a sufficiently attractive vacancy meet.

The value of search for an unemployed worker, U, is given by equation (4). An employer with a vacancy faces a similar problem. Let c denote the flow cost of recruiting a worker to fill a vacancy and let η denote the frequency with which an employer encounters workers seeking employment. Clearly, the value of holding the job vacant V, the expected present value of future profit, solves the following analogue of equation (4).

$$V = \frac{-c}{r+\eta} + \frac{\eta}{r+\eta} \int \max\{V, J\} dG(J) \tag{7}$$

[4] The value of continued search is not stationary if (a) the horizon is finite, (b) payoff realizations reveals information about its distribution to the searcher, or (c) the environment is non-stationary. Although all of these cases have been studied in the partial equilibrium literature, (c) is the principal source of non-stationarity considered in the equilibrium literature.

[5] See Devine and Kiefer (1991), Wolpin (1995), and Neumann (1997) for reviews of the empirical literature on unemployment and job spell duration and postspell wage rate that apply this theory.

where J represents the value of filling the job and G denotes its distribution across workers.

In the case of *transferable utility,*[6] the total value of the match to the pair is the sum of the shares received by its partners, i.e.,

$$W + J = X. \tag{8}$$

In market equilibrium, the value of each share will be determined by the wage outcome. To satisfy individual rationality, the share received by each partner must exceed the forgone option of continued search. Regarding these values (U, V) as the "threat point", a general solution to this problem is one that gives some fraction β of the net surplus $X - (U + V)$ to the worker and the remainder to the firm, i.e.

$$W - U = \beta(X - U - V), \ \beta \in [0, 1]. \tag{9}$$

A necessary and sufficient condition for the formation of a match under individual rationality and transferable utility is that $X - (U + V) \geq 0$.

Match rents are divided between firm and worker by the wage rule. Wage determination is a major issue in the context of search equilibrium modelling. Unlike competitive theory without friction, an existing match will always command quasi-rents *ex post* because it is costly in time and resources for either party in the pair to seek the next best alternative. Given the existence of these quasi-rents, the "market wage" is not unique in this environment. Any division that satisfies individual rationality is a formal possibility. However, the most common specification found in the literature is the assumption that rents are divided with the worker's share β regarded as a free parameter. Possible justifications as well as alternatives assumptions about wage determination are considered subsequently.

2.3 Matching technology

A *matching technology*, like a production technology, is a description of the relation between inputs, search and recruiting activity, and the output of the matching process, the flow rate at which unemployed worker and vacant jobs form new job-worker matches. Because an employer joins a worker when a worker joins an employer to form a match, an "adding up" condition holds that needs to be made explicit: The flow rate at which unemployed workers

[6] The case of non-transferable utility in a search equilibrium context, the marriage problem, is not reviewed here although there are a number of excellent recent papers available on the topic. For example, see Burdett and Coles (1997), Burdett and Wright (1998), and Smith (1997).

meet vacancies is identically equal to the flow rate at which employers with vacant jobs meet applicants. Formally, the assumption that each searching worker meets prospective employers at frequency λ implies that the expected aggregate rate at which unemployed workers meet vacant jobs is equal to λu where u denotes the measure of unemployed workers. Similarly, the assumption that each vacancy is visited by workers at frequency η implies that the aggregate rate at which vacancies meet applicants is ηv where v represent the measure of vacancies. These two flows are identically equal. Obviously, since the vacancy and unemployment pair (v, u) can be anything, the identity $\lambda u \equiv \eta v$ requires that the arrival frequencies are functions of the measures of participation, u and v.

The general solution to this problem found in the literature is to invoke a *matching function*, denoted as $m(v, u)$, which characterizes the aggregate meeting rate. Then

$$\lambda u \equiv m(v, u) \equiv \eta v \tag{10}$$

implies that each of the two meeting rates functions, $\lambda = m(v, u)/u$ and $\eta = m(v, u)/v$, represent the average rate at which unemployed workers and vacancies meet potential partners. The matching function summarizes all the details of the meeting process is a manner analogous to the way an aggregate production function summarizes a production process. Namely, it is the "output" of the meeting process expressed as a function of its inputs, as reflected in the measures of agents of each type participating in the process.[7]

At the micro level, different matching functions can be derived from specific specifications of the meeting process. For example, if each agent on one side of the market has all the telephone numbers of unmatched agents on the other side and each makes contact by phoning a number chosen at random from time to time, then one can show that the meeting function in continuous time takes the linear form $m = fu + gv$ where f represents the calling frequency of the typical unemployed worker and g is the average frequency with which employers with vacancies make calls. Formally, fu calls are made per period of length dt by the searching workers. As the expected number of calls made by workers per employer with a vacancy per period of length dt is $fudt/v$ and the actual number of calls received by any one employer is a Poisson random variable, the probability that a particular employer is not called during the interval is $e^{-\frac{fu}{v}dt}$. Hence, the number of employers who receive one or more calls is $v(1 - e^{-\frac{fu}{v}dt})$. Analogously, the number of workers who receive at least one call from some employer with a vacancy is $u(1 - e^{-\frac{gv}{u}dt})$. The aggregate contact rate per unit period is the sum of these two numbers

[7] Although the matching function was not stated and estimated explicitly until the early 80s, it features in earlier search models, most notably Phelps (1970) and Bowden (1978).

divided by the period length dt. Taking the limit as the latter tends to zero, one obtains

$$\begin{aligned}&\lim_{dt\to 0}\left\{\frac{v(1-e^{-\frac{fu}{v}dt})+u(1-e^{-\frac{gv}{u}dt})}{dt}\right\}\\ =\ &\lim_{dt\to 0}\left\{fue^{-\frac{fu}{v}dt}+gve^{-\frac{gv}{u}dt}\right\}=fu+gv\end{aligned}$$

provided that $(u,v)>0$.

However, if the telephone book includes all agents on the other side of the market, matched and unmatched, then the aggregate rate at which *unmatched agents* of the two types meet is $m=fuv/k+gvu/l$, where l and k represent the total number of worker and jobs, since v/k is the probability that a randomly selected employer will have a vacancy and u/l is the probability that a randomly selected worker will be unemployed. Although in both cases the aggregate matching rate is increasing (and continuous) in its arguments, a condition any reasonable meeting process should have, the matching function is homogenous of degree one in the first case but exhibits increasing returns in the second in the sense that doubling the numbers of participates of the two type quadruples the meeting rate.[8]

The most common specification in the applied literature is the log linear or 'Cobb-Douglas' matching function with constant returns, i.e.

$$m(v,u)=m_0v^{1-\alpha}u^{\alpha},$$

with $0<\alpha<1$. Although the nature of the specific matching process that might generate such a function is not known, Pissarides (1986) and Blanchard and Diamond (1989) provide empirical justification for this form. Indeed, their results suggest that an elasticity parameter α is in the neighborhood of one-half is consist with aggregate data.[9]

2.4 Search equilibrium

As participation is voluntary, a full definition of search equilibrium must specify the measures of search participants, u and v, as well as the participation values for worker and employer, U and V. The form of the matching

[8] Diamond and Maskin (1979) call the first the "linear" case and the second the "quadratic" case.

[9] However, more recent research based on micro panel data suggest important elements of increasing returns to scale. For example, see Warren (1996), Coles and Smith (1994), and Munich, Svenjar and Terrel (1997).

function and the specific model of unemployment and vacancy determination have important consequences for the existence and uniqueness of search equilibrium because they characterize the ways in which the decisions to participate strategically interact. For example, suppose that search participation is "essential" in the meeting process in the usual sense of production theory, i.e., $m(0,u) = m(v,0) = 0$. No participation by all, $(v,u) = (0,0)$, is always an equilibrium in the non-cooperative game theoretic sense in this case. Formally, $v = 0 \Rightarrow \lambda = m(0,u)/u = 0 \Rightarrow U = (b-c)/r \leq b/r$ for all $u > 0$ by equation (4) and $u = 0 \Rightarrow \eta = m(v,0)/v = 0 \Rightarrow V = -c/r \leq 0$ for all $v > 0$ by equation (7). As a worker's value of not participating is b/r and an employer's value of not participating is 0, no one on the either side of the market has an incentive to participate if no one participates on the other side of the market.

With the matching technology determining the arrival rates, the equations (2), (7), (9), and (10) imply that the value of unemployed search U and job vacancy V solve the system of equations

$$U = \frac{b - a + \frac{m(v,u)}{u}\beta \int \max\{X, U+V\} dF(X)}{r + \frac{m(v,u)}{u}} \tag{11}$$

and

$$V = \frac{-c + \frac{m(v,u)}{v}(1-\beta) \int \max\{X, U+V\} dF(X)}{r + \frac{m(v,u)}{v}}. \tag{12}$$

Because the matching function, $m(v,u)$, is increasing in both arguments, the unemployed workers and vacant jobs are *complements* in the sense that an increase in the measure of one type increases the value of participation for agents on the other side of the market. On the same side of the market there is a *congestion effect*, i.e., a greater number of the same type reduces their own value of participation. This effect, however, requires that the meeting rate $m(v,u)/u$ for unemployed workers and $m(v,u)/v$, decreases with that type's participation measure; a property possessed by all concave constant returns matching functions.

There are several alternative approaches to modelling the specifics of unemployment and vacancy determination. The simplest supposes an unlimited supply of participants of both types. In this case, worker and jobs enter until the numbers are such that each is indifferent between participating and not participating. Because workers earn b per period when not participating and there are no pure profit opportunities for employers, a *search equilibrium unemployment-vacancy pair* (u,v) is any solution to $U = b/r$ and $V = 0$. As a consequence of the properties of (11) and (12), if there are constant returns in the matching technology only a non-participation equilibrium or a continuum of equilibria $(u, v) = (0, 0)$ exists since both $m(u, v)/u$ and $m(u, v)/v$ are function of the ratio u/v. Indeed, a solution pair $(u^*, v^*) > 0$ exists which is also "stable" in the sense that the value of participation diminish with the participation measures if and only if the matching technology exhibits decreasing returns to scale in eventually.[10] Finally, if the matching function exhibits increasing returns for small values of u and v as well, multiple equilibria are possible.

These results follow from other strong specification assumptions. The value of a match, X, is independent of the number of matches is one. The supply of participants is unbounded for any positive value of participation is another. Partly motivated by these observation but also by the empirical fact that the aggregate labor supply does not change very much over time for endogenous reasons, much of the labor market literature has adopted the assumption that the total labor supply is constant. Given the fixed size, a natural normalization is unity, so u becomes the unemployment rate and v the vacancy rate. The assumption of unlimited entry of vacancies has, however, been retained, so the equilibrium level of vacancies v solves the no profit condition $V = 0$. A steady condition determines unemployment u. Once again, in most of the existing analyses X is independent of employment, i.e., there are no diminishing returns to scale in production. Constant returns to scale in the matching technology, in the sense that $m(v, u)$ is homogenous of degree one, is also typically assumed.

A third approach to the determination of v and u is that adopted by Diamond (1982b) and Blanchard and Diamond (1989). They fix both the labor force size and the total number of jobs. The equilibrium pair in this case is found from two steady state conditions, one for unemployment and one for vacancies.

3 Equilibrium Unemployment

In this section we review the theory of equilibrium unemployment for labor markets characterized by frictions summarized in Pissarides (1990). The flow of newly created jobs is the outcome of a matching process in which both workers and employers participate. Wages are determined by a generalized Nash bargain after worker and employer meet. The labor force size is constant but firms create job vacancies until any incremental profit is exhausted.

[10] We leave a proof of this assertion as an excercise for the reader.

3.1 Exogenous job destruction

Consider first a simple environment characterized by identical workers and employers with no uncertainty about match product, a matching function that determines the flow of new jobs created, and a fixed job destruction rate. (See Pissarides, 1990, for a detailed analysis of this case.) Jobs are created at the rate $m(v,u)$, where, as before, v is the vacancy rate and u the unemployment rate. The matching function is increasing, concave, and homogenous of degree 1. All jobs have the same productivity denoted as p. Employer and worker negotiate a wage when they meet and subsequently produce until an idiosyncratic shock arrives that destroys the job match. At separation, the firm leaves the market and the worker joins unemployment to look for another job. The arrival rate of the idiosyncratic shock is a constant δ.

The evolution of unemployment is given by

$$\dot{u} = \delta(1-u) - m(v,u). \tag{13}$$

Under the assumption that the matching technology exhibits constant returns, this equation has a unique stable steady state solution for every vacancy rate v, i.e.,

$$u = \frac{\delta}{\delta + m(v/u, 1)} = \frac{\delta}{\delta + \lambda(\theta)}. \tag{14}$$

where the vacancy to unemployment ratio $\theta = v/u$ signals market *tightness* and $\lambda(\theta) = m(v/u, 1)$ represents the unemployment spell hazard.

When equation (14) is drawn in vacancy-unemployment space it generates a *Beveridge curve*, a negative relation between vacancies and unemployment. It is convex to the origin by the properties of the matching function. One can also express the relationship in terms borrowed from the empirical literature on job creation and job destruction. Divide both job creation and job separation flows by employment $1-u$ to generate the *job creation rate* $m(v,u)/(1-u)$ and the *job destruction rate* δ. Equation (14) gives the unemployment rate that equates the endogenous job creation rate with the constant job destruction rate.

Equilibrium unemployment depends on the parameters of the model through the dependence of endogenous market tightness on them. Market tightness, in turn, uniquely determines the duration of unemployment. Equilibrium market tightness is obtained from profit maximization given the wage bargain. The firm maintains a job vacancy by incurring a flow cost of recruiting a worker c. Applications arrive at the rate $m(v,u)/v$ which we denote by

$$\eta(\theta) = \frac{m(\theta, 1)}{\theta} = \frac{\lambda(\theta)}{\theta}. \tag{15}$$

Since $\lambda(\theta)$ is increasing and concave, $\eta'(\theta) < 0$ with elasticity $1-\theta\lambda'(\theta)/\lambda(\theta)$ between zero and one.

When any unemployed worker and employer with a vacancy meet, wage bargaining takes place. The outcome is a wage w that divides the quasi-rents associated with a match between worker and employer. Given an arbitrary wage w, the associated value of a filled job to the employer, J, solves the asset pricing equation

$$rJ = p - w - \delta J \tag{16}$$

where p represents the output of the match. As the value is lost in the event of destruction, equation (16) is equivalent to the continuous time Bellman equation

$$J = \frac{1}{1+rdt}\left((p-w)\,dt + (1-\delta dt)J\right)$$

where δdt represent the probability of job destruction during any sufficiently small time interval of length dt. Analogously, the value of the job to the worker satisfies

$$rW = w - \delta(W-U). \tag{17}$$

The difference in the workers case is that job destruction generates unemployed search which has value U.[11]

We seek the generalized Nash bargaining wage outcome, that which solves

$$\begin{aligned} w &= \arg\max(W-U)^{\beta}(J-V)^{1-\beta} \\ &= \arg\max\left(\frac{w-rU}{r+\delta}\right)^{\beta}\left(\frac{p-w-(r+\delta)V}{r+\delta}\right)^{1-\beta} \end{aligned} \tag{18}$$

where V equals the employers value of holding the job vacant and U is the value of continued search, i.e., the values of the agents outside options. The maximization implies that the worker's share of match surplus is the constant β, formally

$$W - U = \beta(W + J - U - V). \tag{19}$$

Substitution from (16) and (17) into (19) gives the implied wage equation

$$w = rU + \beta[p - rU - (r+\delta)V] \tag{20}$$

[11] Alternatively, if δ were interpreted as the worker "death" rate, the employer would have a vacant job in the event of separation which has value V. The simplifying assumption here is that all separations are the consequence of job destruction rather than worker quitting behavior. One can easily generalized the value equations to account for both reasons for separation.

The wage outcome compensates the worker for the loss of his return to unemployment rU, and pays in addition a fraction β of the net flow return of the match, which is the match product p, net of the worker's and firm's reservation incomes, $r(U+V)$, and the lost value of the job site in the event of destruction, δV.

The value of unemployment U satisfies

$$rU = b + \lambda(\theta)(W - U). \tag{21}$$

where here b represents the value of leisure or home production foregone when employed, less any cost of search denoted as a above but suppressed here. Similarly, the value of a new vacancy, V, solves

$$rV = -c + \eta(\theta)(J - V) \tag{22}$$

where here c represents the flow cost of recruiting a worker expressed as a fraction of the worker's productivity once employed. Profit maximization and free entry require that all rents from new vacancy creation are exhausted, i.e., the *job creation condition*

$$V = 0 \tag{23}$$

holds.

A steady state *search equilibrium* for this simple economy is vector (u, w, θ, V, U) that satisfies (14), (20), (22), (21), and (23). Substitution from (16) and (23) into (22) yields the alternative form for the job creation condition

$$\frac{c}{\eta(\theta)} = \frac{p - w}{r + \delta}. \tag{24}$$

Note that equation (24) is a dynamic demand for labor condition. As the expected duration of the vacancy is $1/\eta(\theta)$, the condition requires that the expected hiring cost equal the present discounted value of the difference between the future flows of marginal product and wage payments where the discount rate is the sum of the interest and job destruction rates. Because equations (22), (21), (19) and (23) imply

$$rV = -c + \frac{(1-\beta)(rU - b)}{\beta\theta} = 0,$$

substitution into (20) for rU gives the equilibrium *wage equation*

$$w = (1-\beta)b + \beta(p + c\theta). \tag{25}$$

The wage increases with the unemployment benefit, job productivity, and market tightness. Finally, the equilibrium is fully described by the wage and market tightness pair (w, θ) which satisfy equations (24) and (25).

The two equilibrium conditions have useful descriptive properties shown in Figure 1. They intersect only once, hence equilibrium is unique. An increase in the worker's unemployment income b shifts the wage curve up and so increases wages and reduces market tightness. An increase in the worker's share parameter β has similar effects. In contrast, an increase in match product p shifts the job creation condition up, increasing both wages and market tightness. More turbulence in the labor market in the sense of an increase in the arrival rate of negative reallocation shocks, δ, shifts the job creation condition down reducing both wages and tightness.

Given the solution for tightness obtained from Figure 1, we can now draw the Beveridge diagram to derive equilibrium unemployment and vacancies as illustrated in Figure 2. With fixed productivity, the solution for θ is independent of unemployment, so equilibrium θ in the Beveridge diagram is shown as a straight line through the origin. Call it the job creation condition. Equilibrium unemployment is at the point where the job creation condition intersects the Beveridge curve.

Following on from our previous analysis, an increase in either the worker's share parameter or unemployment income rotates the job creation line down in Figure 2 and so increases equilibrium unemployment. A higher job productivity rotates it up and reduces unemployment. Higher arrival rate of idiosyncratic shocks shifts the Beveridge curve out and rotates the job creation line down, so it increases unemployment but has ambiguous effects on vacancies.

The contrasting results that we obtained for different productivity levels and different arrival rates of idiosyncratic shocks form the basis of much of the discussion about whether changes in unemployment (and hence the business cycle) are driven by aggregate shocks or reallocation shocks. Papers that have explored this contrast include Jackman et al (1989) who find that the rise in UK unemployment in the 1970s and to some extent in the 1980s was not due to aggregate shocks and Abraham and Katz (1986) and Blanchard and Diamond (1989), who attributed cyclical shocks in the United States largely to aggregate shocks, changes in p.

Andolfatto (1996) and Merz (1995) adopt the search equilibrium approach as a characterization of the labor market in a dynamic stochastic general equilibrium macroeconomic model of capital accumulation. In these models, a household sector decides how to allocate current production between consumption and saving as well as how to divide a time endowment between work, search and leisure. See Merz (1997) for a review of the contributions of these hybrid models found in the 'real business cycle' literature. These applications include studies of the trade-off between the insurance benefits provided and the allocation distortions induced by unemployment insurance systems by Andolfatto (1996), Costain (1995), and Valdivia (1995).

3.2 Job and worker flows

Empirical evidence shows that the job destruction flow, δ in the notation of the preceding section, is not constant, especially over business cycle frequencies (see Davis et al, 1996). In this section we generalize the model to variable job destruction flow and derive the equilibrium conditions (see Mortensen and Pissarides, 1994).

The model builds on that of the preceding section by allowing future job productivity to take more than two values. We write job productivity as px where $x \in [0, 1]$ represents the relative value of a job's specific service or product. Suppose that the product or service provided by the match is a irreversible decision made at the time the job is created. An idiosyncratic shock to the productivity of a match, a new value of its relative value x, arrives at a finite constant rate δ and is distributed according to the c.d.f. $F(.)$, assumed independent of previous realizations. Thus, the idiosyncratic shock process to the value of worker product has persistence but no memory. Profit maximization at the time the job is created requires that product or service be of highest relative value, $x = 1$, given that future values are determined by the first order Markov process assumed above. The model of the previous section can now be re-interpreted as one where the support of the distribution of a future idiosyncratic productivity shock is the unit interval.

The value of a filled job with idiosyncratic productivity x is denoted $J(x)$ and satisfies the functional equation

$$rJ(x) = px - w(x) + \delta \left[\int \max \langle J(\tilde{x}), 0 \rangle \, dF(\tilde{x}) - J(x) \right], \tag{26}$$

where the expression on the right accounts for the fact that the employer will end the match if its future expected present value falls below zero. Job creation takes place as before, so condition (23) still holds but where the value of a new vacancy satisfies

$$rV = -c + \eta(\theta)[J(1) - V]. \tag{27}$$

because a new job has initial relative value $x = 1$. The value of a job to the worker when idiosyncratic productivity is x solves the functional equation

$$rW(x) = w(x) + \delta \left[\int \max \langle W(\tilde{x}), U \rangle \, d\tilde{x} - W(x) \right] \tag{28}$$

where the right side reflects that fact that the worker would quit to search were the worker's value of match to fall below the value of unemployment. Finally, the value of unemployed search solves

$$rU = b + \lambda(\theta)[W(1) - U]. \tag{29}$$

The Nash wage bargain is a contingent wage contract defined by

$$w(x) = \arg\max[W(x) - U]^{\beta}[J(x) - V]^{1-\beta}.$$

Because

$$W(x) = U + \beta\,[J(x) + W(x) - U - V] \tag{30}$$

is implied, the wage contract is

$$w(x) = rU + \beta[px - rV - rU]. \tag{31}$$

Given the wage equation (31), both job values, $J(x)$ and $W(x)$, are monotonically increasing in x. Furthermore, $J(x) - V$ is positive if and only if $W(x) - U$ is positive. Therefore the job destruction policy satisfies a reservation property, i.e. a job is destroyed only if its idiosyncratic productivity falls below a critical level R, which satisfies

$$W(R) - U = J(R) - V = 0. \tag{32}$$

Job separations are still equal to total job destruction. With unemployment given by u, total job destruction is given by $\delta(1-u)F(R)$. Therefore, the job destruction rate now is $\delta F(R)$, with R satisfying (32) and, therefore, the evolution of unemployment is given

$$\dot{u} = \delta(1-u)F(R) - m(v,u). \tag{33}$$

In steady state unemployment satisfies

$$u = \frac{\delta F(R)}{\delta F(R) + \lambda(\theta)}, \tag{34}$$

where as before $\theta = v/u$ is market tightness and $\lambda(\theta) = m(\theta, 1)$.

Steady-state *search equilibrium* is a tuple $(u, v, w(x), R, V, U)$ that satisfies the job creation and job destruction conditions (23) and (32), the wage equations (31), the flow equilibrium condition for unemployment, (34), and the values of vacancy and of search unemployment equations, (27) and (29). Equilibrium is again unique, a property that we illustrate with a diagram after derivations of more explicit expressions for the job creation and job destruction conditions.

The job value equation (26) can be rewritten as

$$(r+\delta)J(x) = (1-\beta)(px - rU) + \delta \int_R^1 J(\tilde{x})dF(\tilde{x}) \tag{35}$$

when the wage contract satisfies (31) and the reservation product solves (32). As $J'(x) = (1-\beta)p/(r+\delta)$ by implication and $J(R) = 0$, it follows that

$$J(x) = (1-\beta)\left(\frac{x-R}{r+\delta}\right) p \text{ for all } x. \tag{36}$$

Hence, for the case of $x = R$ equations (32), (35) and (36) imply that the reservation product solves

$$\left(R + \frac{\delta}{r+\delta}\int_R^1 (x-R)dF(x)\right) p = rU. \tag{37}$$

The reservation productivity, pR, falls short of the worker's return to search, the term on the right side, by the option value of continuing the match, the second term on the left side. The option value is positive because the relative value of an existing job may increase in the future.

Equations (23), (27), (30) and (36) imply that the job creation condition can be written as

$$c = (1-\beta)\eta(\theta)\left(\frac{1-R}{r+\delta}\right) p. \tag{38}$$

The relation between the two unknowns implied by the job creation condition is negative because the vacancy hazard, $\eta(\theta)$, decrease with tightness and because at a higher reservation productivity the expected life of a new job is shorter, and so the expected profit from a new job is lower. Fewer job vacancies are created, reducing market tightness, as R increases given the free entry condition. The negative relation between R and θ implied by (38) is indicated in Figure 3 by the job creation curve, JC in the sequel. Finally, the flow return to unemployment search is an increasing linear function of market tightness

$$rU = b + \frac{\beta c\theta}{1-\beta} \tag{39}$$

by virtue of equations (29) and (30). These three equations, (37), (38), and (39), characterize search equilibrium in the model of job creation and job destruction flows.

For given θ, equations (37) and (39) imply that the reservation productivity falls with δ and rises with r, because of the effects that each has on the option value of continuing the job-worker match. It rises with unemployment income b and falls with match product p because of their effects on the forgone relative income when the worker is unemployed. Finally, the reservation productivity increases with market tightness because the expected returns to unemployed search increases with θ. The dependence of the reservation productivity on market tightness is illustrated in Figure 3 by the job destruction curve referred to as JD in the sequel. The equilibrium solution for the pair (R, θ) lies at the unique intersection of the two curves JC and JD drawn in Figure 3.

As an illustration of the new results obtained with endogenous job destruction flow, consider the implications of two parametric shifts, higher unemployment income b and greater market 'turbulence' δ. Because a higher b represents an increase in the opportunity cost of employment, the JD curve shifts up in Figure 3. Equilibrium R rises and equilibrium θ falls in response. Thus, the job destruction rate, which is equal to $\delta F(R)$, unambiguously rises. At a given unemployment rate, job creation falls because the fall in θ implies a fall in vacancies. Because the job creation rate has to increase to match the higher job destruction rate, the steady state employment falls and the unemployment rate rises. We also see this in the Beveridge diagram in Figure 2, where the fall in θ rotates the job creation line down and the rise in R shifts the Beveridge curve out, increasing steady state unemployment. Finally, wages may rise or fall in equilibrium because, on the one hand, wages rise with b but, on the other, they fall with θ. In sum, higher unemployment income implies more job destruction and less job creation.

Consider now the implication of faster arrival of idiosyncratic productivity shocks, an increase in δ. In response, the option value of continuing a match increases so that the reservation productivity must fall given θ from (37), i.e., the JD curve in Figure 3 shifts down. Because higher δ reduces the expected life of a job and so leads to less vacancy creation from (38), the JC curve shifts to the left. The equilibrium reservation productivity R falls but the diagram does not give a clear answer about the change in θ. However by completely differentiating the equilibrium conditions, one can show that θ also falls.

The implications of an increase in δ for unemployment and for the job destruction and job creation rates are not clear-cut. The job destruction rate, $\delta F(R)$, may rise or fall because the direct effect from an increase in δ can in principle be offset by the fall in the equilibrium value of θ. Although the fall in θ initially implies less job creation, the resulting change is unemployment is also ambiguous. In the Beveridge diagram, the fall in θ rotates the job creation line down, but the ambiguity about the change in job destruction introduces an ambiguity about the direction of the shift in the Beveridge curve.

Mortensen and Pissarides (1994), Mortensen (1994b) and Cole and Rogerson (1996) all find that an extension of the model that regards p as a stochastic process characterizing an aggregate shock are consistent with the time series characteristics of job creation and job destruction series reported by Davis et. al (1996).

3.3 Social efficiency

There are two offsetting external effects of the decision to participate in any labor market with search friction. Because the expected time required to fill a vacancy, $v/m(v,u)$, is increasing in v, the marginal vacancy has a congestion effect on other competing vacancies. However, the marginal vacancy also decreases the time a worker can expect to spend searching for a job, $u/m(v,u)$. For a class of related models, Hosios (1990) establishes that these two externalities just offset one another in the sense that search equilibrium is socially efficient if and only if the matching function is homogeneous of degree one and the worker's share of surplus β is equal the elasticity of the matching function with respect to unemployment. This same condition is both necessary and sufficient in the case of the Mortensen and Pissarides (1994) model as well as its extension by Merz (1995).

Hosios' efficiency condition characterize the solution to a utilitarian social planner's problem. Namely, in the case of a linearly homogenous matching function

$$\beta = \frac{um_u(v,u)}{m(v,u)} = 1 - \frac{\theta\lambda'(\theta)}{\lambda(\theta)} \tag{40}$$

is necessary and sufficient for a search equilibrium to maximize the expected present value of future aggregate income where $\lambda(\theta) = m(\theta, 1) = m(v,u)/u = \theta\eta(\theta)$ is the unemployment hazard rate. We provide a derivation of this result below.

Recall that new matches produce at rate p, subsequently productivity shocks arrive at rate δ, the new value is px with x distributed $F(x)$, and only jobs that have realization $x \geq R$ continue. Hence, gross market output rate evolves according to

$$y' = \lambda(\theta)dtup + \delta dt(1-u)p\int_R^1 xdF(x) + [1-\delta dt]y \tag{41}$$

where $y' = y(t+dt)$ is the market output rate at the end of the time interval $[t, t+dt)$ and $y = y(t)$ is rate at the beginning of the interval. As before, the path of unemployment solves

$$u' = \delta dtF(R)(1-u) + [1-\lambda(\theta)dt]u \tag{42}$$

where $u' = u(t+dt)$ and $u = u(t)$. The aggregate net income flow during the interval is $(y + bu - cv)dt$ after account is taken of unemployment income and recruiting costs.

The utilitarian social planner's problem is to choose a future time path for the decision variables, the reservation product and tightness pair (R, θ), that maximizes the expected present value of future aggregate income, defined as market output net of recruiting costs plus unemployment income. The value function for the problem solves the Bellman equation

$$L(y,u) = \max_{R,\theta} \left\{ \left(\frac{1}{1+rdt} \right) [(y + bu - c\theta u)dt + L(y', u')] \right\}. \qquad (43)$$

As the right side is a contraction map for all $rdt > 0$, a unique solution exists for the value function $L(y, u)$. Because the right sides of (41) and (42) are linear in y and current income is also linear in both given the decision variables, the contraction maps the set of linear functions, a compact metric space, to itself. Hence, the solution is necessarily linear by the contraction map theorem.

The first order necessary conditions required of an optimal choice of (R, θ) are

$$\begin{aligned} [L_u - L_y pR]\, \delta dt F'(R)(1-u) &= 0 \\ [(L_y p - L_u)\, \lambda'(\theta) - c]\, udt &= 0 \end{aligned}$$

where L_y and L_u are the constant partial derivatives of the value function. Note that the second order conditions are satisfied at any solution if $m(v, u)$ is concave in v and homogenous of degree one since then $\lambda''(\theta) < 0$. By the envelope theorem, the partial derivatives of the value function must satisfy

$$\begin{aligned} L_y &= \left(\frac{1}{1+rdt} \right) [dt + L_y(1 - dt\delta)] \\ L_u &= \left(\frac{1}{1+rdt} \right) \left[\begin{array}{c} bdt + L_u \left(1 - \delta dt F(R) - \lambda(\theta)dt\right) \\ +L_y \left(\lambda(\theta) - \delta \int_R^1 x dF(x) \right) pdt \end{array} \right]. \end{aligned}$$

Because the first equation in each group together imply $L_y = 1/(r+\delta)$ and $L_u = pRL_y = pR/(r+\delta)$, appropriate substitution into the remaining two equations yields the necessary and sufficient conditions characterizing the stationary optimal decision pair:

$$c = \lambda'(\theta) \frac{p(1-R)}{r+\delta} \qquad (44)$$

and

$$\left(R+\int_R^\infty (x-R)dF(x)\right)p = b + c\left(\frac{\lambda(\theta)-\theta\lambda'(\theta)}{\theta\lambda'(\theta)}\right)\theta. \tag{45}$$

By inspection, these necessary and sufficient conditions for social optimality are equivalent to the equilibrium conditions, equations (37)-(39), given $\lambda(\theta) \equiv \theta\eta(\theta)$, if and only if the employer's share of match surplus is equal the elasticity of the matching function with respect to vacancies, i.e., the Hosios condition (40) holds.

Because the participation externalities on the two sides of the labor market are off setting, the job creation and job destruction flows can be either too high or too low when social optimality fails. To see this, consider the dependence of the decentralized equilibrium on the share parameter β. In Figure 3, the JC curve shifts left and the JD curve shifts up in response to an increase in β. Hence, the equilibrium value of θ unambiguously falls but the effect on R is not clear-cut. A complete differentiation of the equilibrium conditions, however, establishes that R reaches a maximum when β satisfies the Hosios condition (40). Hence, job destruction is always too low when efficiency fails but market tightness is too low if β is above its efficient level and too high if it is below it.

4 Alternative Models of Wage Determination

The search equilibrium framework is useful in the study of the unemployment effects of alternative wage determination mechanisms (see Mortensen (1989)). Although most of the literature on equilibrium unemployment incorporates the Nash bargaining assumption, many of the most salient implications of the theory are robust to the wage mechanism specifically assumed. In this section we explore the differing implications of several alternative models of wage determination. We ask how the unemployment equilibria obtained in each case compare with the efficient outcome, that chosen by a social planner. First, we study a "competitive" mechanism that ensures efficiency and subsequently consider the implications of a "monopoly union" wage model, an "insider-outsider" model and an "efficiency wage" model.

4.1 Competitive search equilibrium

Moen (1997) and Shimer (1995) construct and analyze closely related wage formation models that generate the socially optimal match surplus sharing rule characterized by the Hosios condition, equation (40). As demonstrated by Greenwald and Stiglitz (1988) and by Mortensen and Wright (1995), the same solution can also be viewed as the outcome of competition among third party market makers who offer unemployed workers and employers with vacant jobs a trade-off between future income when matched and matching delay. Perfect competition among match makers in this economy generates implicit prices for expected waiting times and these prices provide the appropriate incentives for worker and employer participation in the matching process in the sense that the marginal social return is equal to the perceived private return of each agent.

Suppose that each of potentially many middleman offers a particular element (β, θ) in an available set of sharing rule and waiting time pairs, denoted as Ω. Given that each employer and worker can freely choose to participate in any one of the markets organized by these middlemen, the values of holding a vacancy and of searching for employment solve

$$rV = \max_{(\beta,\theta)\in\Omega} \left\{ (1-\beta)\eta(\theta) \left(\frac{1-R}{r+\delta} \right) p - c \right\} \tag{46}$$

and

$$rU = \max_{(\beta,\theta)\in\Omega} \left\{ b + \beta\lambda(\theta) \left(\frac{1-R}{r+\delta} \right) p \right\} \tag{47}$$

because the surplus sharing rule requires

$$\frac{1-R}{r+\delta} = \frac{J(1)-V}{1-\beta} = \frac{W(1)-U}{\beta}.$$

Free entry, $V = 0$, implies that

$$(1-\beta) \left(\frac{1-R}{r+\delta} \right) p = \frac{c}{\eta(\theta)} \tag{48}$$

must hold in every sub-market.. Finally, the reservation product R is the submarket characterized by a given pair (β, θ) must solve

$$\left(R + \frac{\delta}{r+\delta} \int_R^\infty (x-R)dF(x) \right) p = rU = b + \frac{\beta c \theta}{(1-\beta)}. \tag{49}$$

In competitive equilibrium in this environment, no market maker earns pure profit. Out of equilibrium, however, a market maker can profit by charging an arbitrarily small fee provided that the sub-market characteristics offered $(\beta', \theta') \notin \Omega$ attract both employers and workers.[12] To do so, both of the following inequalities must hold and one holds strictly:

$$(1-\beta')\eta(\theta') \left(\frac{1-R}{r+\delta} \right) p - c \geq rV$$

[12]For simplicity, we assume that middlemen incur no costs.

$$b + \beta' \lambda(\theta') \left(\frac{1-R}{r+\delta} \right) p \geq rU.$$

To eliminate such pure profit potential, then, every element (β, θ) of the equilibrium set Ω must satisfy the tangency condition

$$\left. \frac{d\beta}{d\theta} \right|_V = -\frac{\lambda(\theta) - \theta\lambda'(\theta)}{\theta\lambda(\theta)}(1-\beta) = -\frac{\lambda'(\theta)}{\lambda(\theta)}\beta = \left. \frac{d\beta}{d\theta} \right|_U \tag{50}$$

given the definition $\theta\eta(\theta) = \lambda(\theta)$. In other words, for any member of the equilibrium set of sub-markets, the rate at which an unemployed worker who participates is willing to exchange market tightness for a share of surplus once matched must equal the rate at which any participating employers with a vacant job is willing to trade the two.

Since the zero profit condition for match making, equation (50), is equivalent to the Hosios condition (40) for a socially efficient search equilibrium, Moen (1997) calls any pair (Ω, R) that satisfy (49), (50), and $V = 0$ a *competitive search equilibrium.* When workers and employers are respectively identical as assumed here, Ω is a singleton pair determined by the unique solution to (48) and (49) and the associated equilibrium worker's share of match surplus is that which solves (50).

One interpretation of the tale told in this section is that the general inefficiency of search equilibrium is due to incomplete markets. In particular, in a complete market model such as that just described, waiting times are appropriately priced, search externalities are internalized, and the overall market equilibrium is Pareto optimal. As it turns out, Pareto optimality implies social efficiency given the linear preferences assumed. Obviously, wages determined by bargaining or many other mechanisms are not likely to yield this outcome.

4.2 Monopoly union

In many labor markets, terms of employment are determined by collective bargaining agreements. The monopoly union formulation represents the standard approach to modeling wage formation in this context. In this Stackelberg game of wage and employment determination, the union first sets the wage and then employers respond by determining employment. The wage determination mechanism can be placed in a search equilibrium framework by supposing that employers create and destroy jobs given a wage contract that specifies the share β of match surplus obtained by workers.[13]

[13] The worker's share of match rents is set as the parameter of a contingent wage contract rather than a fixed wage level to quarantee job destruction that is individually rational from the view of both worker and employer.

Pissarides (1990) shows that a union would set the worker's share β at its efficient value given by the Hosios condition (40) were all of its members unemployed. However, if the union acts in the interest of employed workers instead, the share exceeds the social optimum, but still the union does not fully exploit market power by appropriating the entire match surplus. Thus, an "insider-outsider" conflict between members of the union exists. Employed workers, the "insiders", want the union to choose higher wages than unemployed workers, the "outsiders".

The proof is simple. If a union were to represent unemployed workers, it would choose β to maximize the equilibrium return to search, which can be written as

$$rU = \left(R + \frac{\delta}{r+\delta} \int_R^1 (x-R)dF(x) \right) p \tag{51}$$

by equations (37) and (39). As the right hand side is increasing in R, the optimal choice maximizes the reservation product. As noted above, a complete differentiation of the equation system composed of (37) - (39) implies that the equilibrium value of R is a concave function of β which is maximized when the worker's share of match surplus satisfies the Hosios condition, equation (40).

In practice, unions represent employed worker. Suppose that the union is a democracy and the median voter is employed in some job with match product $\widehat{x} > R$. As equations (23), (30), and (36) imply

$$W(\widehat{x}) = U + \beta p \left(\frac{\widehat{x} - R}{r+\delta} \right),$$

the share β that maximizes $W(\widehat{x})$ is defined by

$$\widehat{\beta} = \arg \max_{\beta \in \{0,1\}} \left\{ U + \beta p \left(\frac{\widehat{x} - R}{r+\delta} \right) \right\}. \tag{52}$$

Equation (51) implies

$$r\frac{\partial U}{\partial \beta} = p \left(\frac{r + \delta F(R)}{r+\delta} \right) \frac{\partial R}{\partial \beta},$$

so the first order condition for a interior solution to the optimization problem is

$$\frac{\partial W(\widehat{x})}{\partial \beta} = p \left(\frac{\widehat{x} - R}{r+\delta} \right) + \frac{p}{r} \left(\frac{r(1-\beta) + \delta F(R)}{r+\delta} \right) \frac{\partial R}{\partial \beta} = 0. \tag{53}$$

Hence, if more than half of all members are employed, $\widehat{x} > R$ for the median voter, then $\frac{\partial R}{\partial \beta} < 0$ at $\beta = \widehat{\beta}$. But, we have already noted that R decreases with β only for values above the efficient share.

Because the associated reservation product is less than the social optimal, the incidence of unemployment, $\delta F(R)$, is also less than that for a competitive search equilibrium. However, the expected duration of an unemployment spell, $1/\lambda(\theta)$ is longer than in the competitive case. As these two effects of a higher worker's share on the steady state unemployment rate are offsetting, the qualitative relationship between the competitive and monopoly unemployment rates is unclear. Interestingly, unemployment duration is longer and incidence is lower in European countries than in the US. As European labor markets are also more unionized, the model may have explanatory power for these differences.

4.3 Strategic bilateral bargaining

The match surplus sharing rule characterized by the generalized Nash bargaining outcome has also been interpreted as a solution to a strategic bargaining game. For example, consider the following simple setting. Imagine that both on meeting and after a productivity shock is realized for a continuing match, worker and employer engage in a bargaining game obeying the following rules: The worker is allowed to make a wage demand with probability β. With complementary probability $1-\beta$, the employer makes a wage offer. After either a wage offer or demand is made, the other party accepts or rejects. In the event of rejection, negotiation stops and worker and employer both search for an alternative partner.

Given the rules of this simple game played under conditions of complete information, the dominant strategy is to demand or offer a wage equal to that needed to make the other party just indifferent between accepting or not. Given that rejection implies search, a worker will demand the entire match surplus $S(x) \equiv J(x) + W(x) - U - V$ while the employer offers a future income stream equivalent to the value of unemployment, U. The expected outcome for the worker in a continuing match is

$$\begin{aligned} W(x) &= \beta\left(U + S(x)\right) + (1-\beta)U \\ &= U + \beta\left[J(x) + W(x) - U - V\right] \end{aligned} \tag{54}$$

for all x. These bargaining outcomes yield the same wage contract as that derived for the generalized Nash bargaining model with worker bargaining power parameter equal to β.

In Rubinstein's (1982) model of strategic bargaining, the negotiation process can't end in the event of rejection given any strictly positive match surplus, i.e., no positive surplus can be "left on the table". Furthermore, the identity of the party making the initial offer is determined by the flip of fair coin and the parties alternate roles in subsequent negotiation rounds. Rubinstein and Wolinsky (1985) show that the symmetric case, $\beta = 1/2$, approximates the unique solution to the bargaining game played by the Rubinstein rules if worker and employer can costlessly search while negotiating and either switches bargaining partners when search generates an alternative.

Binmore, Rubinstein and Wolinsky (1986) and Wolinsky (1987), argue that other outcomes can also obtain in general. For example, in the extreme case in which the arrival rate of an outside bargaining option during the negotiation process is zero for both parties and the worker makes the wage demand with probability β in each round, the expected present value of earning prior to negotiation is

$$W(x) = \max \langle \ \beta \left[J(x) + W(x) \right], U \rangle \tag{55}$$

In other words, the worker receives a fixed share of match value, if that amount exceeds the value of unemployed search, and the value of unemployment otherwise, where the share β is the worker's probability of setting the terms. This outcome is the consequence of the fact that the party who realizes the right to set terms in any round of the negotiation will demand the entire value of the match less whatever must be transferred *ex ante* to induce the other party to participate in the match. Although this outcome will generate the same job destruction rule, the decision to invest in job creation is distorted even if the share parameter β satisfied the Hosios condition.

4.4 Rent sharing with turnover costs

The differences in preference over wages of the unemployed and those of the employed in the monopoly union case is an example of what Lindbeck and Snower (1988) have termed "insider-outsider" conflict. The conflict arises in that case because the costs of finding a job are sunk for an employed worker but not for an unemployed worker. Insider-outsider conflict also arises in hold-up problems of the type pointed out by Grout (1984).[14] Hiring and firing costs, which we interpret as the fixed costs of job creation and job destruction respectively, motivate hold-ups.

Assume that the legal and economic environments are such that the firm is liable for initial hiring and training costs and for subsequent firing costs in the event of job destruction but worker and employer are able to precommit to an enforceable wage contract which determines terms of employment contingent on future events when they form a match. In general, the wage structure

[14] Caballero and Hammour (1994, 1996) and Acemoglu (1996) discuss hold-up problems in search and matching problems.

that arises as a Nash bargaining solution has two-tiers under these conditions with the property that the worker shares the initial hiring cost and prepay expected firing cost by accepting a lower initial wage but later enjoys a higher wage.[15] The lower first tier wage reflects the fact that hiring costs are directly relevant to the decision to accept a match and that the possibility of incurring firing costs in the future affects the value the employer places on the match. The higher second tier wage applies at some later tenure when firing costs are directly relevant to continuation decision and when separation without renegotiation would otherwise violate the interests of both parties.

Because the second tier wage is generally higher than the first tier, a worker, once "inside", has an incentive to default on the original two-tier agreement by demanding to renegotiate immediately after being hired. Indeed, as Lindbeck and Snower (1988) argues, evidence suggests that workers once employed do take actions designed to extract the quasi-rents created by recruiting, hiring, and firing costs, i.e. a "hold-up" problem exists. The employment effects of such behavior can be studied in the search equilibrium framework by comparing equilibrium conditions with and without the two-tiered wage structure.

We first introduce fixed hiring and firing costs and derive the two-tier wage structure that they induce. Suppose the employer is obliged to pay hiring cost C in order to begin production and firing costs T at time of job destruction. The former can be viewed as the sum of application, processing, and training costs, forms of match specific investment. Examples of the latter include the costs implicit in mandated employment protection legislation and in experience rated unemployment insurance taxes. However, a pure severance transfer, a payment to the worker by the employer, is not included in T. For reasons pointed out by Lazear (1990) and Burda (1992), the equilibrium values of the relevant decision variable pair (R, θ) is unaffected by a severance payment although the magnitude of the payment will effect the wages over the duration of the match.

Let the subscript $i = 0$ index the initial wage and values of a job and employment under the terms of the two-tier contract. For simplicity, we assume that the second tier of the wage contract applies to all matches once an idiosyncratic shock to match productivity occurs.[16] The value of a new job

[15] Alternatively, the worker could 'buy' rights to the job by making an initial transfer to the employer. This kind of side payment is ruled out here on empirical grounds. One theoretical explanation for why side payments of this form are not observed is that no employer can precommit to a future employment duration under the 'employment at will' doctrine.

[16] MacLeod and Malcomson (1993) argue that the initial wage agreement will be renegotiated only if not doing so would result in an inefficient separation, i.e., the destruction

match to the employer under these assumptions and notational conventions is

$$rJ_0 = p - w_0 + \delta\left[\int_R^1 J(\tilde{x})dF(\tilde{x}) - F(R)T - J_0\right] \tag{56}$$

while the initial value of the match to the worker is

$$rW_0 = w_0 + \delta\left[\int_R^1 W(s)dF(s) + F(R)U - W_0\right] \tag{57}$$

where $J(x)$ and $W(x)$ are the values of continuing the match to worker and employer under the second tier contract. These value functions solve the analogous functional equations

$$rJ(x) = px - w(x) + \delta\left[\int_R^1 J(\tilde{x})dF(\tilde{x}) - F(R)T - J(x)\right] \tag{58}$$

and

$$rW(x) = w(x) + \delta\left[\int_R^1 W(s)dF(s) + F(R)U - W(x)\right] \tag{59}$$

where $w(x)$ represents the productivity contingent second tier wage contract.

The value of unemployment satisfies

$$rU = b + \lambda(\theta)[W_0 - U] \tag{60}$$

and, because the creation cost C is incurred when the match forms, the value of a vacant job solves

$$rV = -c + \eta(\theta)\left[J_0 - V - C\right] = 0 \tag{61}$$

given free entry. Finally, the job destruction condition requires a future expected loss greater in expected present value than the cost of termination, i.e.,

$$J(R) = -T. \tag{62}$$

The initial first tier wage rate satisfies the Nash condition

$$w_0 = \arg\max(W_0 - U)^\beta(J_0 - C - V)^{1-\beta}. \tag{63}$$

The employer's threat point includes hiring costs in the initial bargaining problem because they are not yet incurred when bargaining takes place and will not be incurred unless the employer can expect to cover them with future

of the job when the sum of both partners future income given continuation exceeds the firing cost. The contract we study has the same two-tier feature, yields the identical job creation and job destruction decision, and is much easier to characterize. None the less, our contract generates a much more "flexible" wage than theirs.

income. As the first order condition requires $\beta(J_0 - C - V) = (1-\beta)(W_0 - U)$, the initial "outsider" wage implied by equations (56) and (57) is

$$w_0 = \beta(p - (r+\delta)C - \delta T) + (1-\beta)rU. \tag{64}$$

given $V = 0$.

In the subsequent bargaining problem, the employer's threat point does not include hiring costs, those are sunk. Instead, the threat point is the negative of the firing cost which would have to be paid were the job destroyed as reflected in the job destruction condition (62). Hence, the second tier wage contract solves

$$w_1(x) = \arg\max (W(x) - U)^{\beta}(J(x) + T)^{1-\beta}. \tag{65}$$

Here, the first order condition requires $\beta(J_0(x) + T) = (1-\beta)(W - U)$. Equivalently, the "insider" wage paid after an idiosyncratic shock is

$$w_1(x) = \beta(px + rT) + (1-\beta)rU] \tag{66}$$

from equations (58) and (59). Thus, hiring and firing costs reduce the initial wage, because they reduce the ex ante value of the match but, given the value of search U, hiring costs do not influence the insider wage because they are sunk whereas firing costs increase it, because they represent a legal commitment that employer must pay in the event of a layoff.

As this definition of R, equation (62), and the form of the continuing wage contract specified in equation (66) imply that the solution to (58) takes the form

$$J(x) = (1-\beta)p\left(\frac{x - R}{r+\delta}\right) - T, \tag{67a}$$

the reservation product solves the following generalization of equation (37):

$$\left(R + \frac{\delta}{r+\delta}\int_R^1 [x - R]dF(x)\right)p + rT = rU. \tag{68}$$

The option value of continuation is augmented by the foregone interest on the firing cost in the generalization, an implicit return attributable to continuing the match one more period.

As the job creation condition, that implied by $V = 0$, can be written

$$c = \eta(\theta)\left[J_0 - C\right],$$

the following generalization of equation (38) holds

$$\frac{c}{\eta(\theta)} + (1-\beta)(C+T) = (1-\beta)\left(\frac{1-R}{r+\delta}\right)p. \tag{69}$$

The employer's share of the total return on a new job-worker match now has to cover the employer's share of all three types of costs, recruiting, which is all paid by the employer and is sunk at the initial bargaining date, plus hiring and firing, both of which are shared with the worker. Finally, the return to search is

$$rU = b + \frac{\beta c}{1-\beta}\theta \tag{70}$$

as before.

An *rent sharing search equilibrium* is a two-tier wage contract $\{w_0, w_1(x)\}$, a tightness-reservation pair (θ, R), and a value of unemployed search U that solves (64), (66), (68), (69) and (70). Once the pair (θ, R) is determined, unemployment equilibrium is obtained as before using the Beveridge equation (34).

By reducing θ for given R an increase in job creation cost C shifts the JC curve to the left in Figure 3. Equilibrium reservation productivity (and job destruction) and market tightness (job creation) both fall in response. Market tightness decreases because fewer vacancies are created when job hiring and training costs are higher. Job destruction decreases because a decrease in market tightness reduces the worker's outside options and so reducing wages. An increase in job destruction cost T shifts the JC curve to the left and in the JD curve down in Figure 3. Job destruction falls for both reasons: Since it is now more expensive to destroy jobs, fewer are destroyed and since fewer jobs are created, wages are lower and fewer jobs are destroyed as a secondary consequence. Because jobs live longer given the reduction of R, the net effect of an increase in destruction cost on job creation is ambiguous.

4.5 Insider wage

As already noted, hold-up problems arise in the absence of a two-tier wage structure. Furthermore, new workers, outsiders, once hired have an *ex post* incentive to renege on the two-tier structure by demanding the higher insider wage. Indeed, some claim that the two-tier structure is infeasible as a consequence. In this section, we look at the implications of a pure insider equilibrium, that which obtains when the second tier wage contract applies initially as well as subsequent to any shock to match productivity.

Given that the initial wage is determined by the continuing wage bargaining outcome, the initial values of a match are given by the values of the match at $x = 1$, i.e.,

$$W_0 = W(1) \text{ and } J_0 = J(1) \tag{71}$$

replace equations (56) and (57). In this case, the wage rate now solves (66) in all jobs. Because turnover costs are not shared with workers in the insider case, one can show that the free entry condition $V = 0$ implies

$$\frac{c}{\eta(\theta)} + C + T = (1-\beta)\left(\frac{1-R}{r+\delta}\right)p. \tag{72}$$

Consequently, market tightness is less at given reservation productivity than it would be under a two-tier contract. Although the job destruction condition is the same as before

$$\left(R + \frac{\delta}{r+\delta}\int_R^1 [x-R]dF(x)\right)p + rT = rU, \tag{73}$$

now the return to search is more responsive to market tightness than it was in the two-tier case, i.e.,

$$rU = b + \frac{\beta}{1-\beta}\left(c\theta + (C+T)\lambda(\theta)\right) \tag{74}$$

After substituting from this expression into (66), one obtains the insider equilibrium wage contract

$$w(x) = (1-\beta)b + \beta[px + rT + c\theta + (C+T)\lambda(\theta)]. \tag{75}$$

A comparison with equations (64) and (66) reveals that all workers earn more at a given level of market tightness θ when insiders impose a uniform wage. This fact is the essence of the hold-up problem, by forcing the employer to bear the whole of the job creation and job destruction costs, that firm's employed workers gain. However, in equilibrium all employers collectively have less incentive to create new jobs.

An *insider search equilibrium* is a wage function $w(x)$, a reservation-tightness pair (R, θ), and a value of unemployed search U that solves (72), (73), (74), and (75). Given the results that we have already discussed, the implications of switching from the two-tier wage structure to the pure insider equilibrium is to increase R (and job destruction) at every given value of θ and to reduce θ (and job creation) at every given value of R. In Figure 3, these effects can be represented by shifts in the JD curve up and the JC curve left, reducing equilibrium market tightness but having ambiguous effects on the reservation productivity. Because of the fall in tightness, unemployment durations are higher in the insider equilibrium but the unemployment incidence effect is unclear.

4.6 Efficiency wage

The idea that the wage is set to motivate worker effort provides the basis for an alternative theory of wage and unemployment determination. In this subsection, a version of the Shapiro and Stiglitz (1984) efficiency wage model is incorporated into the search equilibrium framework. As pointed out by Ramey and Watson (1996), a study of this synthesis suggests that job destruction is excessive when a high wage combined with a threat of dismissal is used to motivate workers.

Workers would rather take leisure on the job than supply effort is the critical assumption. An employer fires any worker found shirking but monitoring is imperfect. To motivate effort, the employer pays an "efficiency wage", one that equates the expected loss in future worker income if caught shirking to the value of leisure enjoyed while shirking. Formally, the wage is set so that the product of the monitoring frequency, denoted as ϕ, and the difference between the value of employment and unemployment to the worker, $W - U$, is equal to the flow cost of effort, which might be regarded as equal to the value of forgone unemployment income b, i.e., the wage solves the

$$\phi(W - U) = b. \tag{76}$$

Since the values of employment with effort and unemployment satisfy

$$rW = w + \delta F(R)\,(U - W)$$

and

$$rU = b + \lambda(\theta)(W - U)$$

given the reservation product-market tightness pair (R, θ), the efficiency wage equation is

$$w = b + \frac{b}{\phi}\left[r + \delta F(R) + \lambda(\theta)\right]. \tag{77}$$

Note that the efficiency wage increases with both R and θ, because a higher wage is required to compensate for a higher layoff frequency and because a higher wage is required when an alternative job is easier to find to maintain the employment surplus needed to motivate worker effort.

As the value of a job with worker productivity x satisfied

$$\begin{aligned} rJ(x) &= px - w + \delta\left[\int_R^1 J(\tilde{x})dF(\tilde{x}) - F(R)T - J(x)\right] \qquad (78) \\ &= px - w + \delta\int_R^1 [J(\tilde{x}) - J(x)]dF(\tilde{x}) - \delta F(R)[J(x) + T], \end{aligned}$$

the solution is

$$J(x)+T=\left(\frac{x-R}{r+\delta}\right)p \tag{79}$$

where by definition the value at the reservation product plus firing cost equal zero, i.e., $J(R)+T=0.$ By setting $x=R$ and by substituting for $J(x)$ using (79), equation (78) implies

$$\begin{aligned}&\left(R+\frac{\delta}{r+\delta}\int_R^1(x-R)dF(x)\right)p+rT \qquad (80)\\ =\quad & w=b+\frac{b}{\phi}\left[r+\delta F(R)+\lambda(\theta)\right].\end{aligned}$$

Given the free entry condition

$$rV=\eta(\theta)\left[J(1)-C\right]-c=0,$$

equation (79) implies

$$\frac{c}{\eta(\theta)}+C+T=\left(\frac{1-R}{r+\delta}\right)p. \tag{81}$$

An *efficiency wage search equilibrium* is a wage w and a reservation product and market tightness pair (R,θ) the satisfy equations (77), (80), and (81). Because workers do not share job creation and job destruction costs in this case as well, the job creation condition is the same as in the insider model. Comparison of the job destruction conditions reveal the differences between the two models. Specifically, the rent obtained by worker in an equilibrium increases with market tightness and turnover costs in the insider case and with the cost of monitoring and the sum of the unemployment an employment hazards in the efficiency wage case.

The possibility of multiple equilibria is an interesting feature of the efficiency wage model. Because the right side of the job destruction condition, equation (80), is increasing in R but the left side is increasing in R as well, there can be multiple values of the reservation product consistent with a given value of the market tightness θ. Because the wage is also increasing in θ, one can show that the JD curve defined by the job destruction condition is upward sloping when the reservation product is outside the support of the distribution match productivity but can slope backward in its interior if the expected gain from shirking, equal to b/ϕ, is large enough. Namely, the locus of reservation product and market tightness pairs that solve (80) can be S-shaped, as represented in Figure 4. As the job creation condition, (81), defines a negatively sloped job creation relation between R and θ represented by the curve JC in the figure, three or even more intersections of the two equilibrium relationships are possible.

The reason for the multiplicity is the positive feedback between the wage and the reservation product. Namely, a higher reservation product requires a higher wage to compensate for the increase in the layoff frequency while a higher wage induces an increase in the reservation product. The necessary and sufficient condition for JD to have a negative slope is that the marginal effect of the reservation product on the wage, $\delta F'(R)\frac{b}{\phi}$, exceeds the marginal effect of an increase on the left side of the job destruction condition, $\frac{r+\delta F(R)}{r+\delta}$. On the support of F, this condition can be ruled out only in the extreme case of a very small ratio of the productivity shock to monitoring frequency.

When multiple equilibria exist, they are Pareto ranked. This fact is the immediate consequence of two observations. First, as all the equilibria lie on the negatively sloped JC curve, the equilibrium with the lowest reservation product most also be the one with the highest degree of market tightness. Second, because the equilibrium worker return to unemployment is $rU = b + \lambda(\theta)\frac{b}{\phi}$, worker value of employment is $W = U + r(\frac{b}{\phi})$, and employer value is $rJ(x) = (x-R)p/(r+\delta) - T$ for all $x \geq R$, all agents prefer the equilibrium with the highest degree of market tightness and lowest reservation product. Of course, this Pareto dominant equilibrium also yields the lowest unemployment rate since both duration and incidence are minimum here on the set of equilibria.

If the equilibrium is unique, the comparative statics of the model are qualitatively similar to the rent sharing models. For example, by shifting the job destruction curve JD to the left, a higher unemployment income decrease equilibrium market tightness and the reservation product. Note, however, that the response of the middle equilibrium point is perverse. For example, unemployment falls in response to an increase in b if three equilibria exist as illustrated in Figure 4.

5 Labor Market Policy Analysis

The equilibrium job creation and job destruction framework reviewed above is a relatively new tool for labor market policy analysis. Still a small and growing literature exists. Millard and Mortensen (1996), Mortensen (1994, 1995), Pissarides (1996), and Coe and Snower (1996) use search equilibrium models to study the effects of payroll and employment taxes and the provision of unemployment insurance (UI) benefits on employment and some cases aggregate income. Ljungqvist and Sargent (1996) look at the interaction effects of more frequent reallocation shocks and a more generous income support policy using a related model. Millard (1994, 1995) studies the effects of employment protection policies modeled as a tax on firing and Mortensen (1996) derives the effects of active labor market policy in the form of a hiring subsidy using related versions of the Mortensen-Pissarides model. Mortensen and Pissarides (1997) use a generalization of their model to study the interaction effects of 'skill biased' technology shocks and both unemployment compensation and employment protection policy.

These authors generally find that policy effects on unemployment can be large in calibrated versions of their models, indeed the effects of policy differences are sufficient to explain observed differences between US and European unemployment rates. Those who raise the issue also find that the forgone output attributable to the disincentive effects of labor market policy can be substantial. The purpose of this section is to illustrate these results in the context of the rent sharing model rather than review the specifics of each application of the approach.

5.1 Modeling labor market policy

Both passive and active labor market policies are incorporated in the extensions considered in the section. Passive policies include unemployment compensation, payroll or employment taxes, and employment protection policy. In the model, we assume that unemployment income is augmented by unemployment compensation equal to $\rho\overline{w}$, where $\overline{w}$ represents the average wage paid by all employers and ρ denotes the *replacement ratio*. Employers are assessed a tax proportional to the wage bill. Let τ represent the *payroll tax rate*. As *employment protection policy* inhibits the employers ability to fire workers, it can be modeled as a contribution to the firing cost, T. Finally, active labor market policy is interpreted as a *hiring subsidy,* a lump sum H paid to the employer when a new worker is hired which is equivalent to a reduction in the private cost of job creation C.

In the sequel we interpret the wage as the earnings the workers receive net of a payroll tax paid by the employer. Since $(1+\tau)w$ replaces w when computing the value of match to an employer but not in the computation of the value of employment to a worker, the net contribution of an increase in the wage to the employer's and worker's values of a match are

$$\frac{\partial J_0}{\partial w}=\frac{\partial J(x)}{\partial w}=\frac{-(1+\tau)}{r+\delta}\text{ and }\frac{\partial W_0}{\partial w}=\frac{\partial W(x)}{\partial w}=\frac{1}{r+\delta}$$

respectively both initially and for any subsequent realization of x by equations (56) - (59). As a consequence, the first order conditions for the bargaining problems (63) and (65) are now

$$(1+\tau)(1-\beta)(W_0-U)=\beta(J_0-V-C)$$

and

$$(1+\tau)(1-\beta)(W(x)-U)=\beta(J(x)+T)$$

respectively. In short, the tax affects the share of match surplus that worker and employer receive. Because $\beta/(1+\tau)$ replaces β and $b+\rho\overline{w}$ replaced b in all the equations that characterize the efficient rent sharing equilibrium, the equilibrium conditions can be expressed as follows:

$$w_0\,(1+\tau)=\beta\,(p-(r+\delta)C-\delta T)+(1-\beta)rU\,(1+\tau)\,. \tag{82}$$

$$w(x)\,(1+\tau)=\beta\,(px+rT)+(1-\beta)rU\,(1+\tau)\,. \tag{83}$$

$$\frac{c}{\eta(\theta)}=(1-\beta)\left[\left(\frac{1-R}{r+\delta}\right)p-(T+C).\right] \tag{84}$$

$$R+\frac{\lambda}{r+\lambda}\int_R^1(x-R)dF(x)+rT=rU\,(1+\tau) \tag{85}$$

$$rU\,(1+\tau)=[b+\rho\overline{w}]\,(1+\tau)+\frac{\beta c\theta}{1-\beta}. \tag{86}$$

The imposition of a payroll tax is equivalent in effect on the equilibrium reservation product and market tightness pair to a proportional increase in the worker's private opportunity cost of employment, unemployment income as represented by $b+\rho\overline{w}$, where the factor of proportionality is the payroll tax rate. This outcome is a consequence of the fact that the bargaining solution is sensitive to the effects of the wage choice on the size of the match surplus that the parties share. In short, they set the wage so as to minimize the distortionary effects of the tax given relative bargaining power.[17]

To close the model, we need the following expression for the average wage paid in equilibrium

$$\begin{aligned}(1+\tau)\overline{w} &= (1+\tau)\left(F(R)w_0+\int_R^1 w(x)dF(x)\right)\\ &= \beta\left[(p-(r+\delta)(C+T))\,F(R)+p\int_R^1 xdF(x)+rT\right]\\ &\quad +(1-\beta)rU(1+\tau)\end{aligned} \tag{87}$$

[17]In the US where the UI benefit is proportional to earnings in the preceeding employment spell, the division of match surplus is similarly effected by the replacement ratio. It does not appear in these equation because a common benefit is assumed which depends on the average but not the individual worker's wage.

one implied by the fact that the steady state fraction of new matches, those for which $x = 1$, is $F(R)$ and the fact $F(x) - F(R)$ represents the steady state fraction of matches with idiosyncratic productivity x or less given $x < 1$.

For the purpose of policy analysis, we are also interested in both the steady state unemployment rate,

$$u = \frac{\delta F(R)}{\delta F(R) + m(\theta)}, \tag{88}$$

and the steady state aggregate income net of recruiting and hiring costs, which is

$$y = p\left(F(R) + \int_R^\infty x dF(x)\right)(1-u) + (b - c\theta - C\lambda(\theta))\,u. \tag{89}$$

The latter represents a measure of overall welfare, one that does not necessarily move in the same direction as employment.

5.2 The qualitative effects of policy

The qualitative comparative static effects of changes in policy parameters can be derived using the two equilibrium relationships and these outcome measures. As both unemployment compensation and a payroll tax increase the effective "supply price" of labor, their effects are qualitatively the same as an increase in unemployment income b. Namely, an increase in either ρ or τ shifts the JD curve in Figure 3 up and to the left but does not directly effect the JC curve. The equilibrium responses are a decrease in tightness and an increase in the reservation product, both of which induce a rise in unemployment. A hiring subsidy shifts the JC curve rightward by reducing the private cost of job creation C but has no effect on the JD curve. Market tightness increases but the reservation product rises reflecting the fact that jobs will have shorter lives when they are easier to create. Because unemployment duration decreases but incidence increases in response, the net impact on unemployment is ambiguous. A firing tax has the opposite effect on job creation and decreases the reservation product given tightness, i.e., the JC curve shifts left and the JD curve shifts down. Although the net effect is again unclear, an increase in T decreases unemployment when incidence falls by proportionately more duration increases.

Because aggregate income is not generally monotone in the unemployment rate, there is no general prediction about the directions of any marginal change in a policy parameter given that all policy parameters are positive. However, if the equilibrium with no policy were the solution to the social planner's problem, as is the case when the worker's share of surplus satisfies the Hosios condition, then a small value of any policy parameters would be associated with a lower level of future income because all of the policies induce distortions.

5.3 The quantitative effects of policy

Indeterminate direction of effects on both unemployment and income reflect the multiple channels of influence on equilibrium outcomes incorporated in the model. Which effect dominates is a quantitative question. Computing responses for specific functional forms and parameter values is a feasible first step in providing quantitative answers, one pursued in several papers in the literature. To illustrate the nature of their results, we report the outcome of similar computational experiments for the rent sharing case.

A matching function of the Cobb-Douglas form is assumed, i.e., $\ln(\lambda(\theta)) = \alpha \ln(\theta)$ where α is the elasticity with respect to vacancies. The distribution of shocks is assumed to be uniform on the support $[\gamma, 1]$, i.e., $F(x) = (x-\gamma)/(1-\gamma)$. The base line parameters used in the computations are reported in Table 1. The policy parameters are chosen to reflect values in the US case, productivity in a new job is normalized at unity, the elasticity of the matching function is consistent with the Blanchard and Diamond (1990) and Pissarides (1986) estimates, the workers' share of match surplus is set to satisfy the Hosios condition for social efficiency, and the costs of recruiting and hiring a worker are consistent with survey results reported by Hamermesh (1993). Finally, unemployment income b and the lower support of the distribution of market product γ are chosen so that the implied steady state unemployment rate is 6.5% and average duration of an unemployment spell is one quarter, numbers that reflect the average experience in the US. over the past twenty years.

Table 1: Baseline Parameter Values: Efficient Rent Sharing

New Job Productivity: $p = 1$
Interest rate: $r = 0.02$ per quarter
Matching elasticity: $\alpha = 0.5$
Recruiting cost: $c\theta/m(\theta, 1) = 0.3$ per worker
Training cost: $C = 0.3$ per worker
Productivity shock frequency: $\delta = 0.1$
Minimum match product: $\gamma = 0.64$ per quarter
Value of leisure: $b = 0.35$ per quarter
Worker's rent share: $\beta = 1 - \alpha = 0.5$
UI benefit replacement ratio: $\rho = 0.2$
Payroll tax rate: $\tau = 0.2$

The quantitative implications of variation in the payroll tax rate and UI benefit replacement ratio on unemployment and aggregate income are illustrated in Tables 2. The signs of the effects on unemployment are obviously consistent with the known qualitative implications of the model and aggregate income responds in the same direction as employment in these cases. The first panel clearly suggests that the quantitative magnitude of employment effects of these two forms of passive policy are large enough to provide an explanation for the observed cross OECD country variation in the unemployment rate. In particular, replacement ratios and payroll tax rates in the order of 35% are not uncommon in Europe. Were the US. rates both in that range, the computations imply that the US unemployment rate too would be of European magnitude.

Table 2: Effects of the Payroll Tax (τ) and UI Benefit (ρ)
Rent Sharing Wage Contract

a. Unemployment Rate (percent)					
	$\tau = 0.0$	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.3$	$\tau = 0.4$
$\rho = 0.0$	4.5	4.7	4.9	5.1	5.4
$\rho = 0.1$	5.1	5.3	5.6	5.9	6.3
$\rho = 0.2$	5.8	6.1	6.5	7.0	7.5
$\rho = 0.3$	6.8	7.4	8.0	8.7	9.7
$\rho = 0.4$	8.5	9.4	10.5	12.1	14.6
b. Income (percent of $p = 1$)					
$\rho = 0.0$	90.5	90.5	90.5	90.5	90.5
$\rho = 0.1$	90.5	90.5	90.5	90.4	90.4
$\rho = 0.2$	90.4	90.4	90.3	90.2	90.0
$\rho = 0.3$	90.2	90.1	89.9	89.6	89.2
$\rho = 0.4$	89.7	89.4	88.9	88.1	86.9

Although the efficiency loss of an increase in either the unemployment compensation benefit replacement ratio or the payroll tax rate is not all that large in a neighborhood of the baseline values, they become very significant at higher tax and replacement rates, particularly as the two increase together. Although the US. would lose only about 1% = 100(90.5-89.6)/90.3 of its income per participant in the labor force were both the replacement ratio and the payroll tax rate raised from roughly 20% to 30%, it would lose about 4% = 100(90.5-86.9)/90.3 of income were they both raised to 40%. Indeed, these numbers imply that liberal unemployment benefit and high payroll tax policies may have a important impact on the average standard of living in Europe relative to the US under the hypothesis that the model describes difference between the performance of the two economies induced by known policy parameter differences.

The tables also suggest that the two policies have increasing and complementary effects in the sense that a larger value of either the payroll tax rate or the replacement ratio contributes positively to both its own marginal effect and to the marginal effect of the other. As emphasized by Coe and Snower (1996), this property implies that a smaller joint policy reform designed to reduce both the payroll tax and the replacement ratio may be far more effective that a large reduction in just one of the two.

The unemployment and income effect of a firing tax T and hiring subsidy H are reported in Tables 3a and 3b. For reference, the tax or subsidy increment assumed, 0.3, is approximately equal to one month's average output per worker.

Table 3: Effects of the Firing Cost (T) and Hiring Subsidy (H)
Rent Sharing Wage Contract

a. Unemployment Rate (percent)					
	$H = 0.0$	$H = 0.25$	$H = 0.5$	$H = 0.75$	$H = 1.0$
$T = 0.0$	6.5	7.0	7.9	7.9	8.3
$T = 0.25$	6.0	6.5	7.4	7.4	7.9
$T = 0.5$	5.3	5.9	6.9	6.9	7.4
$T = 0.75$	4.6	5.3	6.4	6.4	6.9
$T = 1.0$	3.9	4.6	5.8	5.8	6.4
b. Income (percent of $p = 1$)					
$T = 0.0$	90.3	90.2	89.9	89.4	88.7
$T = 0.25$	90.2	90.3	90.2	89.9	89.4
$T = 0.5$	89.9	90.2	90.3	90.2	89.9
$T = 0.75$	89.4	89.9	90.2	90.3	90.2
$T = 1.0$	88.7	89.4	89.9	90.2	90.3

There are three observations of interest. First, a employment protection policy lowers unemployment in this experiment while a hiring subsidy increases unemployment. The reason is the same in both cases: The effect on unemployment duration, $1/\lambda(\theta)$, is proportionately smaller than the offsetting effect on unemployment incidence, $\delta F(R)$. Second, the marginal income effect of either a hiring subsidy or a firing tax can be of the *opposite* sign of the marginal employment effect. Finally, as the results along the diagonals of both table imply, a hiring subsidy financed by a severance tax ($H = T$) can lower the unemployment rate with no loss in steady state income although admittedly the effect of such a revenue neutral policy on unemployment is quite small in this simulation.

5.4 A call for research

As stated in the introduction to the section, the computational exercise is meant to be illustrative. As an indication of results that are consistent with the general theory presented in this paper, they demonstrate potentially important implications of the framework. However, more analysis of three kinds is needed. First, how sensitive are these quantitative results to variation in parameter values? Second, how robust are they to alternative specifications of the wage determination process? Finally, econometric research within the framework is required to generate valid information about the reasonable ranges of parameter values and about the actual wage determination process.

6 Wage Posting Games

The idea that employers set the terms of employment while workers choose among available offers is consistent with how many labor economists view the wage setting process. The presence of search friction in the form of incomplete information on the workers side about specific employer offers is a source of monopsony employer power in this setting. Relative to the bilateral bargaining formulation, the model's structure is asymmetric in that it gives the power to set wages to the employer. However, unlike the standard model of static employer monopsony, an employer's market power is constrained by competition with other similar employers over time. The purpose of this section is to review the results drawn from an analysis of a set of models in which employers post wage offers and workers seek the highest offer.

The wage posting approach is consistent with the idea that each employer chooses a particular wage policy, say to be either a "high" or a "low" wage firm. Those that offer high wages are more attractive to outsiders and retain insiders more readily. Facing the same trade off between wage, size and quit rate, some choose the high wage even though profit generated per worker is lower, making up the difference in higher volume. More productive firms find it profitable to acquire more workers by outbidding their less efficient competition. Although 'wage policy' plays no role in formal market models with complete information, the recent work on search equilibrium in wage posting games gives formal content to these ideas.

The models reviewed in the section were also motivated by a purely theoretical question. After the adaptation of optimal stopping theory to the price search problem, Rothschild (1973) asked whether it was possible to derive the distribution of wages that motivate wage search as a market equilibrium phenomena. In particular, does dispersion exist even when all buyers and sellers are respectively identical? Interesting, the answer is a qualified yes. As a consequence, the theory also has substantive content as an explanation for wage differences across observably identical workers and jobs.

6.1 The Diamond paradox

As is well known, non-cooperative price posting under perfect information generates a Bertrand equilibrium, one in which all charge a common competitive equilibrium price, even when the number of competitors is small. Diamond (1971) was the first to solve a fully consistent equilibrium version of the price posting game under imperfect information about offers. He finds that *only the monopoly (monopsony) price* is offered in equilibrium if the price setters are the sellers (buyers) of the good or service in question even when the number of competitors is large. Indeed, Diamond's unique solution to a wage posting game when workers and employers are identical yields the same outcome as a bargaining-matching model in which the employer has all the 'bargaining power' in the sense that the worker's Nash bargaining parameter β is equal to zero. However, because employers capture all the match surplus, there is no non-trivial equilibrium in which workers are willing to participate if the cost of gathering wage information is strictly positive. This result is known as the *Diamond paradox*.

Formally, the structure of the model follows. There is a continuum of active employers represented by the unit interval and a given continuum of potential worker participants represented by $[1, n]$ where n is the measure of workers per firm. Both workers and employers are respectively identical. Each participating worker searches by drawing a sequential random wage sample at frequency λ without recall from the wage offer c.d.f. $F(w)$. Existing job-worker matches dissolve at exogenous rate δ. Under stationary conditions, the value of employment at a job paying wage w, denoted as $W(w)$, and the value of unemployment U given that worker search employers at random solve the following continuous time Bellman equations:

$$rW(w) = w + \delta[U - W(w)] \tag{90}$$

and

$$rU = b - a + \int \max \langle W(w) - U, 0 \rangle \, dF(w). \tag{91}$$

where b represents an unemployment income, any unemployment compensation plus the value of time which would otherwise be gone once an employment spell begins, and a is the out-of-pocket cost of search. Since the first equation implies that the value of employment increases with the wage, the optimal acceptance strategy has the reservation property and the reservation wage, the solution to $W(R) = U$, solves the standard reservation price equation

$$R = b - a + \frac{\lambda}{r+\delta} \int_R^\infty [w - R]\, dF(w). \tag{92}$$

As participation in the labor market while not employed requires search activity and because any out-of-pocket cost of search a is avoided when not searching, participation requires $U \geq b/r$. Assuming that this participation condition holds, the potential labor force is fixed and equal to n. Hence, the measure of the stock of searching unemployed u evolves according to

$$\dot{u} = \delta(n-u) - \lambda u S(R), \text{ where } S(R) = \int_{x \geq R} dF(x)$$

is the probability that a randomly searched employer is offering an acceptable wage and $\delta(n-u)$ is the exogenous worker flow from employment to unemployment. Hence, the number of workers who participate in non-trivial equilibrium is

$$u = \frac{\delta n}{\delta + \lambda S(R)}. \tag{93}$$

Again, let p, a positive constant, represent the value of a worker's marginal revenue flow once employed. Given that the set of employers is represented by the unit interval, every employer hires a flow of workers equal to λu provided that her wage offer w is acceptable and losses workers at rate δ so that the steady state employment of a firm offering wage w is $\ell(w) = \lambda u/\delta$ if the offer is acceptable and is 0 if not. Hence, the expected present value of the flow of profits is

$$\pi(w,R,F) = (p-w)\ell(w) = \begin{cases} \frac{\lambda(p-w)n}{\delta+\lambda S(R)} \text{ if } w \geq R \\ \\ 0 \text{ otherwise} \end{cases}. \tag{94}$$

A *wage posting equilibrium* is a reservation wage R, which is optimal chosen by workers in these sense of (92) and taken as given by employers, and a offer distribution F, also taken as given by each employer, such that every wage offered is profit maximizing, i.e.,

$$w = \arg\max\{\pi(w,R,F)\} \text{ for every } w \text{ on the support of } F. \tag{95}$$

In short, an equilibrium is a reservation wage and wage offer distribution pair (R,F) that represents a non-cooperative solution to a game in which each worker chooses his reservation wage given the wage offers of employers and the reservation wage of other workers and each employer chooses her wage offer taking as given the reservation wages of the workers and the offers of other employers.[18]

The Diamond paradox is easily stated: There is no equilibrium in which exchange takes place if the cost of search is strictly positive. Instead, no workers participate. The proof is simple. Given (94), the only solution to (95) is a wage offer equal to the reservation wage no matter what other employers offer simply because a higher wage attracts no additional worker, i.e., a unit mass on $w = R$ is the only candidate for an equilibrium offer distribution F. But given this offer distribution, $R = b - a$ from (92) which implies that the value of search unemployment is less than the value of non-participation if cost of search is positive, i.e. $U = W(R) = (b - a)/r < b/r$ given $a > 0$ from equations (90) and (91). As a corollary, all workers participate and the common equilibrium wage offered by all employers is b if $a = 0$.

6.2 Wage dispersion: Differential costs of search

One implication of Diamond's paradox is that market failure is the inevitable consequence of costly search and market power on one side of the market. Albrecht and Axell (1984) show that this conclusion is a consequence of the assumption that all workers have identical search and opportunity costs of employment provided that some subset of workers can search costlessly. The essence of their argument follows.

Suppose there are two groups of workers, indexed by $i = 0$ and $i = 1$, that face different search costs a_i satisfying $0 = a_0 < a_1$. Assume further, that the opportunity cost of employment, b_i, enjoyed while either searching or not participating is strictly larger for the group with zero out of pocket costs of search, i.e., $b_0 > b_1 = 0$.[19] In this case, the reservation wages of the two types solve

$$R_i = b_i - a_i + \frac{\lambda}{r+\delta}\int_{R_i}^{\infty} [w - R_i]\, dF(w),\ i = 0 \text{ and } 1, \tag{96}$$

and $rU_i = R_i \geq b_i$ is necessary and sufficient for participation by worker type i. One can easily verify, the workers with the lower search cost and higher outside option value are more selective, i.e., $R_0 > R_1$. In steady state, the

[18]The assumption that an employer acts to maximize steady state profit is a simplification. Given that the interest rate r is small relative to the offer arrival rate λ, the wage offer strategies chosen in equilibrium approximate those chosen were one to use the more appropriate expected present value of future profit criterion in the sense that they are the limiting strategies as the ratio r/λ tends to zero.

[19]The assumption that $a_0 = 0$ is critical but $b_1 = 0$ can be regarded as a normalization.

measure of each type who participate as unemployed workers is determined by

$$u_i = \begin{cases} \frac{\delta n_i}{\delta+\lambda S(R_i)} \text{ if } U_i \geq b_i/r \\ 0 \text{ otherwise} \end{cases}, i = 0, 1, \tag{97}$$

where n_i represents the measure of the total labor force of type i workers per firm.

Obviously, only Diamond's outcome is obtains if only one of the two types participate. Given that both types will search when wage offers are high enough, the possibility of an equilibrium in which both types participate needs to be investigated. Because the application flow to every employer is $\lambda(u_0 + u_1)$ per period, both types accept if the offer is R_0 or more, but only type 1 workers accept for wage offers in the interval $[R_1, R_0)$, and those that accept are employed for an average spell of length $1/\delta$, employer profit is

$$\pi(w, R_0, R_1, F) = (p - w)\ell(w) = \begin{cases} \frac{\lambda(p-w)n_0}{\delta+\lambda S(R_0)} + \frac{\lambda(p-w)n_1}{\delta+\lambda S(R_1)} \text{ if } w \geq R_0 \\ \frac{\lambda(p-w)n_1}{\delta+\lambda S(R_1)} \text{ if } R_0 > w \geq R_1 \\ 0 \text{ otherwise} \end{cases}. \tag{98}$$

given that cost of search and opportunity cost of employment assumptions and equation (96) imply $R_0 > R_1$ for any offer distribution F. Hence, a steady state equilibrium with both types participating is a triple (R_0, R_1, F) that satisfies (95) with employer payoffs defined by (98) and reservation wage rates that satisfy (96) provided profits and non-negative and workers of both types are willing to participate.

Because an employer's payoff strictly declines with the wage offered between worker reservation wage rates but jumps up as the wage offered crosses these two critical numbers, a wage offer is profit maximizing only if it is the reservation wage rate of some worker type. Hence, if there is a non-Diamond outcome, some positive fraction of employers must offer R_0 while the others offer $R_1 < R_0$. Of course, the expected profit made by these two groups of employers must be the same by virtue of (95). Because all offers are no less than R_1, i.e., $S(R_1) = 1$, the equal profit condition requires

$$\begin{aligned} \pi(R_0, R_0, R_1, F) &= \frac{\lambda(p - R_0)n_0}{\delta + \lambda q} + \frac{\lambda(p - R_0)n_1}{\delta + \lambda} \\ \pi(R_1, R_0, R_1, F) &= \frac{\lambda(p - R_1)n_1}{\delta + \lambda} \end{aligned}$$

where $q \equiv S(R_0)$ is the fraction of employers who offer the higher wage $w = R_0$ and $1 - q$ is the fraction that offer the lower wage $w = R_1$. Finally equation (96) and the assumption $a_0 = b_1 = 0$ imply that the value of reservation wage rates satisfy

$$R_0 = b_0$$

and

$$R_1 = \frac{\lambda q}{r+\delta}(R_0 - R_1) - a_1$$

provided that $R_1 \geq rU_1 = b_1 = 0$ so that type 1 worker participate.

By substituting from the last two equations back into the equal profit condition, one obtains

$$\frac{(b_0 - R_1)(\delta + \lambda q)}{(b_0 + a_1)(\delta + \lambda)} = \frac{(p - b_0)n_0}{(b_0 + a_1)n_1} = \frac{(r+\delta)(\delta + \lambda q)}{(\delta + \lambda)(r + \delta + \lambda q)}$$

after manipulating terms. As positive profits are required in equilibrium to guarantee employer participation and the last term on the right is strictly increasing in q, a unique fraction of the employers pay the high wage and the remainder offer the low wage if and only if

$$\frac{\delta}{\delta+\lambda} < \frac{(p - b_0)n_0}{(b_0 + a_1)n_1} < \frac{r+\delta}{r+\delta+\lambda}.$$

Because an open set of parameters satisfies these necessary and sufficient conditions, equilibria with distinct wage offers generically exist. Finally, when the left hand inequality does not hold, then $q = 0$ which implies that all employers offer the wage $R_1 = -a_1 < 0$ and none of the workers participate. Similarly, all employers offer a single wage equal to $R_0 = b_0$ and type 1 worker participate if and only if $R_1 = (\lambda b_0 - (r+\delta)a_1)/(r+\delta+\lambda) \geq 0$ when the right inequality fails. Note in passing that a single wage equal to the reservation wage of the high reservation type is always the outcome in the limit with no friction in the sense that offer arrival rate λ is infinite.

6.3 Wage dispersion: More than one offer

Diamond's paradox is also sensitive to the assumption that search is sequential without recall. Burdett and Judd (1983) show that other equilibria also exist when workers are able to compare two or more offers simultaneously. Indeed, *if every searching workers chooses among two or more randomly selected offers, then the outcome of the wage posting game is identical to Bertrand's, i.e., all employers offer $w = p$.* However, if some positive strict fraction receive two or more offers and another fraction receives only one, the unique equilibrium is characterized by a non-degenerate offer distribution. Furthermore, the distribution converges to a point mass on p as the fraction that receive two or more tends to unity and to a point mass on the common reservation wage R as the fraction who receive more than one offer tends to zero. Hence, all intermediate cases lie somewhere midway between the single wage competitive (Bertrand) equilibrium and single wage monopsony (Diamond) equilibrium even though wages are disperse.

Prior to the Burdett-Judd analysis, Butters (1977) proposed and studied a model naturally satisfied the Burdett-Judd comparative shopping condition. In that formulation, employers "advertise" wage offers by sending messages, 'help wanted ads', to workers at random. Each worker chooses among all advertised job offers received within some specified 'period', say a week. Given that offers arrive continuously over the week at frequency λ, the number received is a random Poisson variable. As the fraction of searching worker's who receive one and two or more wage offer during the period satisfy $0 < \lambda e^{-\lambda} < 1$ and $0 < 1 - e^{-\lambda}(1 + \lambda) < 1$ respectively, the only equilibrium offer distribution is disperse. Of course, one can interpret the arrival rate here as the outcome of the recruiting activity by employer in Butters.

The Burdett-Judd comparison shopping condition is satisfied automatically when unemployed workers search sequentially but employed workers search as well because employed workers who find an alternative can compare it with the wage earned on their current job. Mortensen (1990), Burdett (1990) and Burdett and Mortensen (1989,1997) use this fact to develop models in which wage dispersion is an equilibrium phenomena under quite general conditions. Because the solution can be characterized in closed form, its structure can and has been estimated using data on wages and unemployment spell durations. This model and its implications are reviewed in the remainder of the section.

6.4 Search on the job

In order to incorporate on the job search, the duration of a job spell must be explicitly modeled. Let $\delta > 0$ denote the exogenous rate of job turnover. Assume that employed workers receive outside offers at arrival frequency $\lambda_1 > 0$, generally different from the arrival rate of offers conditional on unemployment denoted as $\lambda_0 > 0$. Mortensen and Neumann (1988) have shown that employed workers accept any offer greater than their current wage and the unemployed accept all wage offer in excess of a reservation wage R which in this case solves

$$R - b = (\lambda_0 - \lambda_1) \int_R^\infty \left(\frac{1 - F(x)}{r + \delta + \lambda_1[1 - F(x)]} \right) dx. \qquad (99)$$

Equation (99) is a generalization of the reservation wage equation (92) which holds when search on the job occurs (where out of pocket search costs are ignored for simplicity). Note that when the offer arrival rate is independent of employment status (as well as out of pocket search cost), the reservation wage is simply equal to the unemployment income b. Furthermore, the effects of the interest rate, the turnover rate and the form of the wage offer distribution on the reservation wage depend critically on the difference between the two arrival rates simply because the relative desirability of search unemployed depends on the difference between the two arrival rates. For example, given an improvement in the offer distribution is the sense of first order stochastic dominance (an increase in $1 - F(x)$ for all x), the reservation wage increases (decreases) if the offer arrival rate when unemployed exceeds (is less than) that when employed because search while unemployed (employed) is more efficient.

Since employed workers move from one job to another without an intervening unemployment spell, the equality of worker flows into, $\lambda_0[1 - F(R)]u$, and out of employment, $\delta(n - u)$, yields the steady state unemployment rate

$$u = \frac{\delta n}{\delta + \lambda_0[1 - F(R)]}. \qquad (100)$$

as before where n represents the number of workers per employer and the total labor supply is fixed and normalized at unity.

By equating the flows into and out of employment at each wage offer, steady state employment at each wage can be derived for any offer distribution. The flow into employment at a wage equal to w or less is $\lambda_0[F(w) - F(R)]u$ given that only offers greater than or equal to the reservation wage are acceptable. The worker flow out of the same category is the sum of exogenous turnover plus the flow of quits to jobs offering a higher wage. The latter flow is equal to $(\delta + \lambda_1[1 - F(w)])G(w)(n - u)$ where $G(w)$ is the fraction of workers employed at wage w or less. Hence, the unique steady state distribution of workers over wage rates associated with any offer distribution is

$$G(w) = \frac{\lambda_0[F(w) - F(R)]u}{(\delta + \lambda_1[1 - F(w)])(n - u)} = \frac{\delta[F(w) - F(R)]}{(\delta + \lambda_1[1 - F(w)])\,(1 - F(R))}. \qquad (101)$$

In the case of a differentiable wage offer distribution, the steady measure of workers employed per firm offering a particular wage w, its steady state labor force, is equal to the ratio of the measure of worker earning the wage divided by the measure of firms offering the wage, i.e.,

$$\ell(w|R,F) = \frac{G'(w)(n-u)}{F'(w)} = \frac{n\delta\lambda_0\left(\delta+\lambda_1[1-F(R)]\right)}{\left(\delta+\lambda_0[1-F(R)]\right)\left[\delta+\lambda_1[1-F(w)]\right]^2} \tag{102}$$

provided $F'(w) > 0$ at w. Hence, the steady state profit per firm offering any wage w in the support of F can be written

$$\pi(w|R,F) = (p-w)\ell(w|R,F). \tag{103}$$

A *wage posting equilibrium* is a common maximal profit earned by each employer π, a reservation wage R and a offer distribution F which satisfy equation (92) and

$$\begin{aligned} \pi(w|R,F) &= \pi \text{ for all } w \text{ on the support of } F \\ \pi(w|R,F) &\leq \pi \text{ otherwise.} \end{aligned} \tag{104}$$

As the employer offering the lowest wage loses all of her workers to competitors anyway, the profit maximizing condition implies that the lower support of the offer distribution is the common reservation wage of all the identical workers. Given (102), (103) and $\underline{w} = R$, equation (104) implies a single candidate for the offer distribution function. Namely,

$$F(w) = \left(\frac{\delta+\lambda_1}{\lambda_1}\right)\left[1-\sqrt{\frac{p-w}{p-R}}\right] \text{ for all } w \in [R,\overline{w}] \tag{105}$$

with upper support

$$\overline{w} = p - \frac{\delta^2(p-R)}{(\delta+\lambda_1)^2}.$$

After substituting from (105) into equation (99), one finds that the equilibrium reservation wage can be represented as a weighted average of the unemployment benefit and worker productivity.

$$R = \frac{(\delta+\lambda_1)^2 b + (\lambda_0-\lambda_1)\lambda_1 p}{(\delta+\lambda_1)^2 + (\lambda_0-\lambda_1)\lambda_1}. \tag{106}$$

Finally, note that $R < p$ so that $\pi = \pi(R|R,F) = \pi(w|R,F) > 0$ on the support of the candidate. Because $\ell(w|R,F) = 0$ for all $w < R$ and

$\ell(w|R,F) = \ell(\overline{w}|R,F)$ for all $w > \overline{w}$, the candidate equilibrium wage offer distribution derived above satisfies the profit maximization condition (104), namely $\pi(w|R,F) < \pi$ for all $w \notin [R,\overline{w}]$, and, consequently, is the only equilibrium offer distribution.[20]

The Burdett-Mortensen equilibrium is in between Diamond's equilibrium and Bertrand's in the sense that both are limiting cases generated by the two extreme assumptions about the rate at which employed workers receive offers. Because $\overline{w} \to R \to b$ as $\lambda_1 \to 0$, the support of the equilibrium wage offer distribution converge to a point equal to the reservation wage as the offer arrival rate when employed tends to zero. Because $G(w) \to 0$ for all $w < \overline{w}$ and $\overline{w} \to p$ as $\lambda_1 \to \infty$, equilibrium converges to a degenerate wage distribution with unit mass concentrated on the competitive outcome, wage equal to the value of marginal product, as the friction vanishes in the sense that the offer arrival rate when employed tends to infinite.

Note that the equilibrium distributions of wages offered and earned, F and G, have increasing convex densities that are highly left skewed with mass concentrated to the right toward the competitive wage p when λ_1 is large. Specifically,

$$F'(w) = \left(\frac{\delta + \lambda_1}{\lambda_1}\right)\sqrt{\frac{1}{(p-R)(p-w)}} \tag{107}$$

and

$$G'(w) = \frac{\delta(\delta + \lambda_1)F'(w)}{(\delta + \lambda_1[1 - F(w)])^2} \tag{108}$$

The left skew simply reflects the fact that all wage offers w are less than p but most are concentrated near the competitive wage p, at least when λ_1 is large. From an econometric point of view then, the Burdett-Mortensen model implies that competitive wage theory has a highly asymmetric "error term" with a negative expectation.

6.5 Worker and employer heterogeneity

Mortensen (1990) demonstrates that a mixture of the Burdett-Mortensen and the Albrecht-Axell outcomes is obtained when workers enjoy different unemployment benefits or search costs. In the generalization, wages other than those on the support of reservation wage distribution are offered but still the wage offer support generally is not convex. Mortensen (1990) also shows how differences in employer productivity can contribute to variance in

[20] The possibility of an equilibrium offer distribution with mass points is ruled out in Burdett and Mortensen (1989).

wage offers. An important characteristic of the equilibrium in this case is that more productive employers offer higher wage rates. Burdett and Mortensen (1997) and Bontemps, Robin, and van den Berg (1997) derive closed form offer distributions in the case of continuous distribution of types for both of these cases. These results and their significance are briefly summarized below.

First, consider the case of a continuum of worker types described by a continuous worker supply price c.d.f. $H(b)$. Assume for simplicity that when the arrival rate is independent of employment status, i.e., $\lambda_0 = \lambda_1$ which implies $R(b) = b$ for each type from equation (99). All employers have the same labor productivity p. The only equilibrium wage offer distribution is of the form

$$F(w) = \left(\frac{\delta + \lambda_1}{\lambda_1}\right)\left[1 - \sqrt{\frac{(p-w)H(w)}{(p-\underline{w})H(\underline{w})}}\right] \tag{109}$$

where the lowest wage, $\underline{w}$, is the largest solution to

$$\underline{w} = \arg\max_{w}\{(p-w)H(w)\}, \tag{110}$$

the highest wage, $\overline{w}$, is the largest solution to

$$\frac{(p-\overline{w})H(\overline{w})}{(p-\underline{w})H(\underline{w})} = \left(\frac{\delta}{\delta + \lambda_1}\right)^2$$

and w is in the support of F if and only if

$$w' > w \Rightarrow \; (p-w)H(w) > (p-w')H(w') \text{ for all } w \in (\underline{w}, \overline{w}). \tag{111}$$

As $H(w)$ is the Marshallian market supply curve in this environment, the lower support is simply the monopsony wage by equation (110). Note, that the equilibrium offer distribution still has an increasing density with a left skew although possibly less so in general than in the case of no dispersion in the unemployment benefit.

Next, consider the case of a continuum of employer types described by the continuous employer productivity c.d.f. $J(p)$. Without loss of generality, assume that the lower support $\underline{p}$ is no less than the common reservation wage R. The only equilibrium wage offer distribution is

$$F(\omega(p)) = J(p) \tag{112}$$

where the wage-productivity profile $w = \omega(p)$ is implicitly defined by the first order condition for profit maximization, which is

$$2\lambda_1 F'(w)(p-w) - [\delta + \lambda_1(1 - F(w)] = 0 \tag{113}$$

on the support of F. It can be shown that the only solution must be

$$\omega(p) = p - \int_R^p \left(\frac{\delta + \lambda_1(1 - J(p))}{\delta + \lambda_1(1 - J(x))} \right)^2 dx. \tag{114}$$

Note, the lowest offer is the common reservation wage, i.e., $J(x) = 0$ for all $x \leq \underline{p}$ implies $\omega(\underline{p}) = R$, wage offers increase with productivity, i.e., $\omega'(p) > 0$, but all offers are strictly greater than the worker's reservation wage but less than the value of marginal product and $R < \omega(p) < p$ for all $p > \underline{p}$. In the case of $R > \underline{p}$, employer's with labor product $p < R$ cannot earn a positive profit and therefore don't participate. The equilibrium offer distribution takes the same form with the truncated distribution of productivity $J(p)/[1 - J(R)]$ replacing J in equations (112) and (114).

Obviously, the shape of the offer distribution is influenced by the shape of the distribution of labor productivity in this case. Indeed, because equations (112) and (114) implies

$$F'(w) = \frac{2\lambda_1}{\delta^2 \int_R^p \frac{\delta + \lambda_1(1 - F(w))}{(\delta + \lambda_1(1 - F(x)))^2} dx},$$

the offer density is not generally increasing. Still, as Bontemps, Robin, and van den Berg (1997) show, equation (107) holds in the limit as dispersion in productivity vanishes. Consequently, the theory implies restriction regarding its form, particularly on the shape of its right tail. These are characterized and used to develope an empirical test of the theory by Bontemps, Robin, and van den Berg (1997).

6.6 Structural estimation

What might be called the "first generation" empirical search literature uses the stopping problem to interpret empirical observation on unemployment spell durations and wages earned immediately after such a spell taking the offer distribution as given. This literature is reviewed in detail by Devine and Kiefer (1991), Wolpin (1995), and Neumann (1997). The papers summarized in this subsection represents on going "second generation" literature in which authors exploit the structure of equilibrium search models in the estimation procedures applied.

The literature starts with Wolpin (1987) and Eckstein and Wolpin (1990) who estimate the Albrecht and Axell (1984) model using panel data on unemployment durations and subsequent earnings. Although the model provides an acceptable fit to the duration data, the fit of the wage data is less satisfactory because each point in the support of the wage offer distribution is necessarily the reservation wage of some worker type in the model. As the complexity of the computation of the equilibrium increases rapidly with the number of types, only a small finite number of types could be considered.

The simple Burdett-Mortensen with homogenous workers and employers is consistent with a number of stylized facts: Wage offers generally exceed reservation wages, workers with more experience and tenure earn a higher wage on average, larger firms offer higher wage rates, and quit rates fall with wage offers in cross section. However, in the absence of exogenous worker or employer heterogeneity in labor productivity, the approach implies increasing densities for both the wage offer and wage earned distributions. Specifically both have left tails skewed away from the unique competitive wage for reasonable values of the offer arrival rates. This implication is at odds with wage distributions which have always have long and thick right tails. As a weaker version of the perfectly competitive implication that every worker's wage is equal to her value of marginal product, it is not a problem in principle. In either case, worker and employer heterogeneity are required to explain the observed shape of earning distributions.

Kiefer and Neumann (1993), Koning, Ridder and van den Berg (1995), and Ridder and van den Berg (1995) all estimate the simplest Burdett and Mortensen (1989) version of the model in which all workers and employers are assumed identical using unemployment and job spell durations and wage data drawn from panel data. They do so by assuming that the labor market is segmented by the usual observable indicators of worker and employer heterogeneity — education, experience, occupation and industry. All the structural parameters, e.g. the offer arrival rates and separation rate, are allowed to vary across the sub-market segments but workers and employers within each sub-market are homogenous by assumption. Although the estimated models provide an accurate fit of both unemployment duration and cross section wage data, the implied dispersion in the sequence of wages that an individual can earn over a work life is too narrow, i.e., the implied return to experience is too small.

Bowlus, Kiefer and Neumann (1995, 1997) provide the first estimates of a version of the model in which exogenous heterogeneity in employer productivity is allowed. They assume only a finite number of employer types, each defined by a different value of labor productivity, p, in the formal model. Because the model's solution cannot be expressed in closed form in this case, their maximum likelihood estimation procedure requires the repeated computation of a candidate equilibrium offer distribution F. Consequently, the computational complexity of the approach grows rapidly with the number of employer productivity types allowed in the support of J. Still their approach fits the data with only four or five points of support and yields interesting and useful results. For example, in their second (1997) paper based on US. data drawn from the NLS, their results suggest that the earnings distribution of young whites stochastically dominates that of young blacks among those transiting from higher school to work primarily because blacks are exposed to twice the subsequent job destruction risk. The estimated offer arrival rates are essentially identical for blacks and whites with the arrival rate when unemployed roughly three times larger than when employed.

Bontemps, Robin, and van den Berg (1997) avoid problems of computational complexity by assuming a continuous distribution of employer types. Their approach permits the application of the first order profit maximization condition, equation (113), and the one to one association it and continuity imply between the distribution of wage offers and the distribution of employer productivity, equation (112). Exploiting these continuous relationship, they obtain joint estimates of the offer arrival rate and separation rate parameters, and a non-parametric estimate of the distribution of employer labor productivity using unemployment and job duration data and earnings data drawn from a French panel survey on individual worker histories. After stratifying the data by industry, the model fits well, even though workers in each industry are assumed equally productive, and the fitted wage offer and wage earned distribution satisfy the tail restrictions implied by the theory. As a further check of adequacy, the authors compare the distribution of productivity over firms implied by the wage data with an independent empirical distribution of value added per worker derived from a sample of French firms in each industry considered. They find that the shapes of these two distributions are broadly consistent with one another. Finally, their empirical results suggest that the most productive employers have significant monopoly power and use it by paying wages substantially below value of marginal product while the least productive have almost none and earn little pure profit.

The estimation methodology applied by Bontemps, Robin, and van den Berg is both simple and powerful. As such, a brief sketch is appropriate. First, the authors use a kernel estimator and the data on employed worker earnings to fit the wage distribution G. Conditional on this estimate, call it $\widehat{G}$, and the parameter λ_1/δ, the inverse of equation (101) generates an associated consistent estimate of the offer distribution $\widehat{F}$ and its density. Second, they substitute these estimates in the unemployment and job spell duration likelihood functions and maximize to obtain consistent estimates of the parameters R, λ_0, λ_1, and δ.[21] Third, they use the first order condition,

[21]Since R varies one for one with b given the other parameters from equation (99), the

equation (113), and $\widehat{F}$ to obtain an estimate of the inverse of $\omega(p)$. After inverting and substituting the result back into $\widehat{F}$, the associated estimate of the distribution of labor productivity $\widehat{J}$ is obtained using equation (112). Note, the method avoids the repeated integration required to obtain $\omega(p)$ using the equilibrium equation (114) which is required by a joint maximum likelihood estimation procedure.

7 Wage Posting in a Matching Model

As suggested by the organization of this essay, the literature on search equilibrium approaches to labor market equilibrium analysis has developed along two somewhat different branches. Although the matching approach has found application primarily in the macroeconomic literature on unemployment determination while the wage posting approach has been used in empirical analysis of wage differentials, the separate lives of these two literatures are difficult to explain, especially since several authors have contributed to both. As potential fruit, a graft of the two strands promises a joint theory of wage offers and market tightness in which employers play the active role of both wage setter and job creator.

A synthesis is sketched in this section based on Mortensen (1998). As his approach is inspired by the related model of Acemoglu and Shimer (1997), the principal results presented here are similar to theirs. First, although Diamond's equilibrium generally exists in the synthesis, so does another equilibrium, one both strictly preferred by all agents and stable under competitive rent seeking job creation behavior. Second, wage dispersion can induce endogenous differentials in labor productivity rather than simply reflect exogenous differences as in the extended version of the Burdett-Mortensen model. In our formulation, this result occurs as a consequence of a standard specific human capital partial equilibrium result, an employer offering a higher wage has an greater incentive to make match specific productivity enhancing investments because the future return on the investment is subject to a less quit risk.

7.1 Search and matching

As in the matching model, firms create "job sites" and each is either vacant or filled. In equilibrium, the vacancy measure v is determined by a zero profit free entry condition. The total labor force size is fixed, normalized to

reservation wage rather than the unemployment benefit can be regarded as a structural parameter.

equal n. Each individual worker is either employed or not and the measure of unemployed, u, evolves according to the usual law of motion. As in Burdett-Mortensen model, workers search while employed as well as unemployed and, consequently, the numbers in each category are inputs, along with the vacancy measure, in the matching technology. Specifically, let $m(v, u, 1-u)$ represent the matching function equal in value to the total flow of offers received by workers. It is increasing and concave in the three arguments.

For simplicity, we assume that employed and unemployed workers are perfect substitutes in the matching process, i.e., only their sum matters. Because the total flow of contacts must equal the sum received by both unemployed and employed workers, the offer arrival rate is independent of employment status and an increasing concave function of vacancies, i.e., as $\lambda_0 u + \lambda_1(1-u) = m(v, u, 1-u) = m(v, u+1-u) \equiv \lambda(v)$ for all u

$$\lambda_0 = \lambda_1 = \lambda(v) \tag{115}$$

where $\lambda(v)$ is an increasing and concave function.

Workers behave as in the Burdett-Mortensen model: Unemployed workers accept the first offer no less than the reservation wage R defined by equation (99) and an employed work accepts any offer in excess of that currently earned. The simplifying assumption that employed and unemployed workers are perfect substitutes implies that the reservation wage is exogenous, i.e., $R = b$. Consequently, the steady state unemployment rate is

$$u = \frac{\delta n}{\delta + \lambda(v)[1 - F(b)]}. \tag{116}$$

Finally, the associated steady state distribution of earnings across employed worker is

$$G(w) = \frac{\delta[F(w) - F(b)]}{(\delta + \lambda(v)[1 - F(w)])\,(1 - F(b))} \tag{117}$$

from equations (100), (101), and (115).

7.2 Wage posting

In the matching framework, it is the future expected return to the creation of a vacancy which is the critical profit concept. Wages are set to maximize this return and entry drives it down to recruiting and hiring costs. Formally, the asset value of a vacant job solves

$$rV = \max_{w \geq R} \{\eta(v)\,[u + (1-u)G(w)]\,(J(w) - V) - c\} \tag{118}$$

where $\eta(v) \equiv \lambda(v)/v$ is the average rate at which vacancies are filled and $c > 0$ is the recruiting cost per vacancy. The first term on the left is the expected return to vacancy creation, the product of the rate at which workers are contacted per vacancy ($\eta(v)$), the probability that the worker contacted will accept (unity if unemployed given $w \geq R$ and $G(w)$ if employed where G is the distribution of wage rates across employed workers), and the capital gain associated with converting a vacancy job to a filled one ($J(w) - V$). As employed workers quit when they are offered a higher alternative wage, the expected present value of the future flow of quasi-rents once a worker is hired, $J(w)$, solves

$$rJ(w) = p - w + (\delta + \lambda(v)[1 - F(w)])\,[V - J(w)] \tag{119}$$

where w is the wage offered, p is match product, δ is the exogenous separation rate and $\lambda(v)[1 - F(w)]$ is the expected rate at which an employed worker finds a job paying more than w. Finally, free entry eliminates pure profit in vacancy creation

$$V = 0. \tag{120}$$

Given the reservation wage, which is tied down by $R = b$ in the case under study, a steady state *wage posting search equilibrium* is an unemployment rate u, a vacancy rate v, a wage c.d.f. G, and a wage offer c.d.f. F which satisfies equations (116), (117), and (120) given that every wage in the support of F is a solution to the profit maximization problem formulated on the right side of (118).

Because no employed worker accepts the lowest wage offer and all unemployed workers accept wages at or above the reservation wage, the lower support of the equilibrium offer distribution is $R = b$. As $F(b) = 0$, an appropriate sequence of substitutions from the other equations into (118) yields

$$\frac{c}{\eta(v)} = \delta \max_w \left\{ \frac{(p - w)n}{(r + \delta + \lambda(v)[1 - F(w)])\,(\delta + \lambda(v)[1 - F(w)])} \right\} \tag{121}$$

But note, in the limiting case of $r = 0$ considered by Burdett and Mortensen, this equation simplifies to

$$\frac{c}{\eta(v)} = \delta \max_w \{(p - w)\ell(w|R, F)\} \tag{122}$$

where

$$\ell(w|R, F) = \frac{n}{(\delta + \lambda(v)[1 - F(w)])^2} \tag{123}$$

is the size of an employer's steady state labor force when offering a wage w in the Burdett and Mortensen model for the special case of offer arrival rates independent of employment status from equation (102).[22] Hence, the equilibrium distribution of offers here is the same as in their model, namely

$$F(w) = \left(\frac{\delta + \lambda(v)}{\lambda(v)}\right)\left[1 - \sqrt{\frac{p-w}{p-b}}\right]. \tag{124}$$

In addition, however, the offer arrival rate is endogenously determined as the solution to the following implication of (122) and (124) and the fact that the lower support of F is b

$$cv = \frac{\delta\lambda(v)(p-b)n}{(\delta + \lambda(v))^2}. \tag{125}$$

As $\lambda(v)$ is increasing and concave, exactly two solution exist, one at $v = 0$, and the second strictly positive under the Inada condition $\lambda(0) = 0$, $\lambda'(0) = \infty$, and $\lambda'(\infty) = 0$ in the only interesting case, that in which labor output exceed the opportunity cost of employment, i.e., $p > b$. Of course, these conditions are quite natural given the production function interpretation of the matching function.

Only the positive solution is stable in the sense that the return to vacancy creation exceeds (is less than) the cost for positive values to its (left) right. In short, the a simple entry process starting with positive vacancies will find the positive equilibrium. Finally, note $c \to 0$ implies that the equilibrium vacancy rate $v \to \infty$. Hence, if the matching function $\lambda(v)$ is unbounded, then competitive equilibrium with all workers earning the common wage p is the result in the limit as recruiting costs vanish.

7.3 Endogenous productive heterogeneity

As demonstrated above, more productive employers offer higher wages in equilibrium in an extended version of the Burdett-Mortensen model characterized by an exogenous distribution of labor productivity over employers. Acemoglu and Shimer (1997) show the causality can be reversed in their model. That is, firms that offer higher wages also have an incentive to differentiate themselves by investing in their worker's. In this section we show that the same result also holds in our synthesis of the matching and wage posting approaches simply because higher wage employers enjoy lower quit

[22] However, in the case of a matching model $\delta\ell(w|R, F)$ represents the accumulated flow of future quasi rents per worker hired in steady state.

rates. Hence, both models suggest a parsimonious explanation for the positive correlation between the wage and labor productivity across employers: The correlation may be the consequence of strategic wage competition.

In the extension, let k represent the cost of training a new hire and let worker productivity be an increasing concave function of this investment denoted as $p(k)$. The value equations (118) and (119) can be rewritten as

$$rV = \max_{(w,k)} \{\eta(v)\,[u + (1-u)G(w)]\,(J(w,k) - k - V) - c\} \tag{126}$$

and

$$rJ(w,k) = p(k) - w + (\delta + \lambda(v)[1 - F(w)])\,[V - J(w,k)] \tag{127}$$

to reflect this extension where the maximization with respect to both wage and investment reflect the simultaneous choice of wage offer and training policy adopted by a particular employer. After the appropriate substitution are made from equations (116), (117), and (120), the equilibrium characterization (121) can be rewritten as

$$\frac{c}{\eta(v)} = \delta n \max_{(w,k)\geq(b,0)} \left\{ \frac{p - w - k\,(r + \delta + \lambda(v)[1 - F(w)])}{(r + \delta + \lambda(v)[1 - F(w)])\,(\delta + \lambda(v)[1 - F(w)])} \right\}. \tag{128}$$

In the limiting case of $r = 0$, the equilibrium wage offer distribution and training investment solve

$$\max_{k\geq 0} \left\{ \frac{p(k) - w - k\,(\delta + \lambda(v)[1 - F(w)])}{(\delta + \lambda(v)[1 - F(w)])^2} \right\} = \max_{k\geq 0} \left\{ \frac{p(k) - b - k\,(\delta + \lambda(v))}{(\delta + \lambda(v))^2} \right\} \tag{129}$$

on the support of F given that $\underline{w} = b$ where any equilibrium vacancy rate solves

$$cv = \frac{\delta n \lambda(v)}{(\delta + \lambda(v))^2} \max_{k\geq 0} \left(p(k) - b - k\,(\delta + \lambda(v))\right). \tag{130}$$

Provided that $p(0) > b$, the Inada conditions applied to the matching technology again guarantee a unique stable positive solution for v. Furthermore, because the left side of (129) is strictly increasing in F given w and is strictly decreasing in w, there is a unique increasing function $F(w)$ which satisfied the equation. Hence, the equilibrium in the extended model is unique and wage offers are dispersed.

The only issue that remains is to characterize differences in the investment policy adopted by employers that offer different wage rates. As the investment decision criterion, implicit in the problem defined on the left side of (129), is strictly concave given the assumption that $p(k)$ is increasing and strictly concave, the investment decision has a unique solution. Assume $p'(0) = \infty$ so that investment for an employer who offers wage w, uniquely solves the first order condition

$$p'(k(w)) = \delta + \lambda(v)[1 - F(w)]. \tag{131}$$

The optimal investment policy expressed as a function of an employer's wage offer $k(w)$, that implicitly defined by the first order condition, is increasing ($k'(w) = -\lambda(v)F'(w)/p''(k) > 0$) because offering a higher wage lowers an employer's match separation rate, the effective rate of depreciation on match specific capital.

8 Summary

This paper has a simple message. Search equilibrium approaches to modeling markets characterized by friction in the form of information gathering delay and turnover costs have matured in the past decade. They are now capable of providing a framework for understanding empirical observation on labor reallocation flows and wage dispersion and for generating important new insights into the effects of labor market policy. We look forward to future applications of the approach to many other substantive questions of interest.

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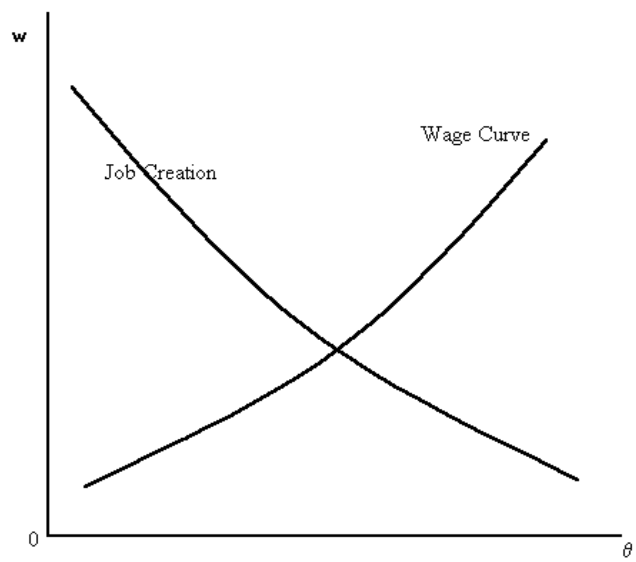

Figure 1: Equilibrium Market Tightness and Wage Rate

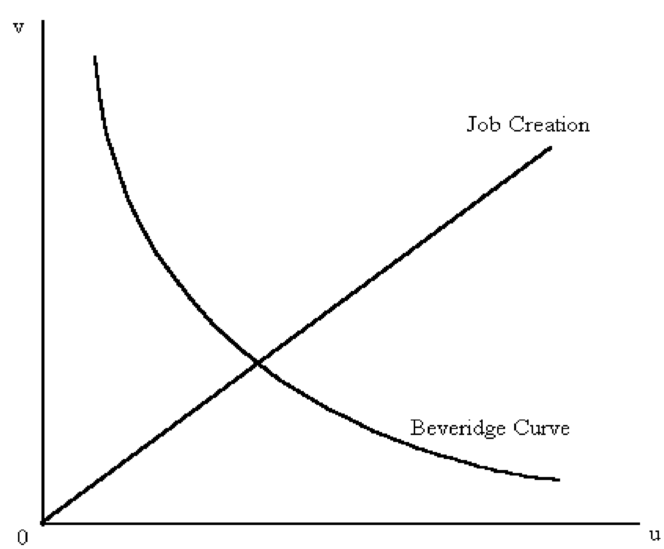

Figure 2: Equilibrium Vacancies and Unemployment

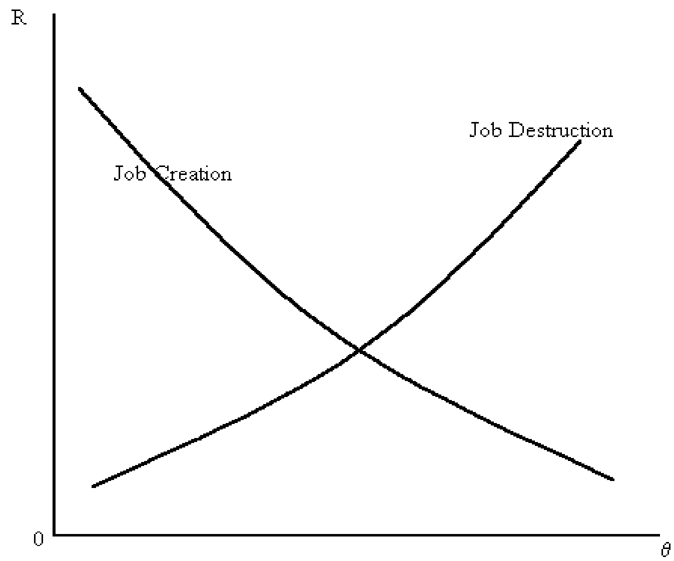

Figure 3: Equilibrium Market Tightness and Reservation Productivity

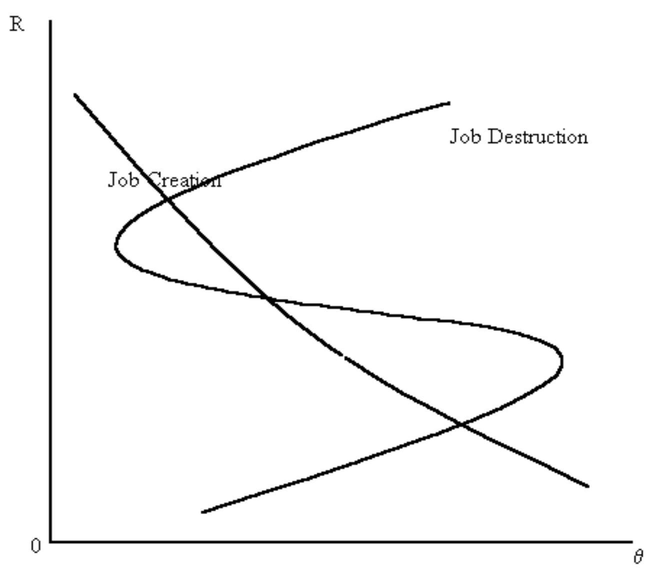

Figure 4: Efficiency Wage Search Equilibria