More on Unemployment and Vacancy Fluctuations[*]

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Abstract

Shimer (2005a) argues that the Mortensen-Pissarides equilibrium search model of unemployment explains only about 10% of the response in the job-finding rate to an aggregate productivity shock. Some of the recent papers inspired by his critique are reviewed and commented on here. Specifically, we suggest that the sole problem is neither the procyclicality of the wage nor the failure to account fully for the opportunity cost of employment. Although an amended version of the model, one that accounts for capital costs and counter cyclic involuntary separations, does much better, it still explains only 40% of the observed volatility of the job-finding rate. Finally, allowing for on-the-job search does not improve the amended model's implications for the amplification of productivity shocks.

Keywords: Labor market search, unemployment and vacancies volatility, job-finding rate, productivity shocks.

JEL Classifications: E24, E32, J41, J63, J64

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1 Introduction

In this paper, we review Shimer's (2005a) critique of the Mortensen-Pissarides (MP) equilibrium search model of unemployment. (See Mortensen and Pissarides (1994,1999a, 1999b) and Pissarides (2000) for an extended development of the model and its implications.) We also add some comments of our own designed to clarify and further the debate generated by Shimer's paper.

Shimer documents the fact that volatility in unemployment is induced primarily by movements in the job-finding rate, the rate of transition from unemployment to employment. He also contends that the magnitude of the response of vacancies and unemployment to labor productivity shocks predicted by the model explains less than 10% of the observed volatility in the job-finding rate in U.S. data given reasonable specification assumptions and parameter values. A principal reason for this lack of explanatory power, he argues, is that the wage, set as the outcome of a bilateral wage bargain, responds to offset almost all the effects of productivity shocks on job creation.

The MP model is designed to account for the fact that it takes time to match jobs and workers. As a consequence of this friction, match specific rents exist when a worker meets a prospective employer. The designers of the original model assume that these rents are shared according to Nash's (1950) axioms with the value of searching for an alternative job serving as the threat point. Hall (2005) argues that any wage in the bargaining set, that consistent with individual rationality for the employer-worker pair, should be regarded as a legitimate equilibrium candidate. He then proceeds to demonstrate that a rigid wage, one not conditioned on the aggregate state, generally exists with the property that it is always in the bargaining set and explains the volatility of unemployment given quantitative specifications of the other elements of the model.

We argue that a flexible wage *per se* is not the principal problem with the model. It is the large difference between labor productivity and the wage implied by the assigned magnitudes of the parameters that is responsible for the lack of amplification of productivity shocks. Even if the wage were rigid, its level must be such that the future flow of quasi-rent attributable to the creation of a new job is very small if the model is to account for the volatility of the vacancy-unemployment ratio observed in the data. As Hagedorn and Manovskii (2005) demonstrate, the model has no problem if the parameters of the wage outcome function are set to match observed average profit rates and wage volatility. Unfortunately, the calibrated opportunity cost of em-

ployment required to explain the observed volatility in the job-finding rate is unrealistically high.

Hall and Milgrom (2005) argue that the outcome of a strategic bargaining game in which the default option is delay rather than unemployed search, along the lines suggested by Binmore, Rubinstein and Wolinsky (1987), is a more realistic specification. They also claim that the amended model substantially improves the degree of amplification implied because the alternative wage rule is less sensitive to productivity shocks. Although indeed the solution to their bargaining game is less volatile, the response to productivity shocks is not that much larger when it is used to determine the wage and Shimer's value of the unemployment payment as the measure of how much the worker benefits from delay is used to calculate the effect.

As an original contribution to the discussion, we augment the model by allowing for capital costs, countercyclical involuntary separations as well as the less cyclical wage implied by the strategic bargaining outcome. After accounting for all these facts, we find that the amended model can explain about 40% of the volatility in the job-finding rate relative to that of productivity observed in Shimer's CPS data. Although our result represent a four fold improvement relative to Shimer's, more amplification seems to be needed.

Some have suggested that accounting for the procyclicality of job-to-job flows can help with amplification. In this paper, we demonstrate that an extension of the model with on-the-job search performs no better; it too explains only 40% of the volatility. The job-finding rate depends primarily on the level of vacancies rather than the vacancy-unemployment ratio when employed worker search, a fact which implies that less amplification of productivity shocks is required to explain the volatility of the job-finding rate. However, the impact of productivity on the return to job creation is also smaller because the job acceptance probability decreases with the job-finding rate and the quit rate increases with the job-finding rate.

The paper also includes two technical contributions. First, we establish that the dynamic extension of the model that Shimer analyzes, one in which productivity shocks are described by a jump process that can be approximated by Brownian motion, has a unique equilibrium solution. Second, we show that the approximation used by Shimer to derive quantitative comparative static results is valid given his estimated productivity process, but not exactly for the reason he states.

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2　The MP Model

In the version of the model Shimer (2005a) considers, all workers and jobs are respectively identical. Furthermore, all agents are risk neutral wealth maximizers. For the reader's sake, we use Shimer's notation for the most part. Specifically, every job-worker match produces market output at the flow rate p. Autocorrelated shocks to p occur from time to time. Hence, the current value of match productivity is an aggregate state variable. The possible dependency of any endogenous variable on the current value of productivity is represented in the notation by using p as a subscript. Following Shimer, we assume that the time sequence $\{p_t\}$ is a jump process characterized by arrival rate λ and a conditional distribution of new values represented by the c.d.f. $F : P \times P \to [0, 1]$ where P is the support of the process.

The opportunity cost of employment to the worker, measured in terms of output, is a non-state-contingent parameter, denoted as z, as is the cost of posting a vacancy, c. As all matches are identical, the flow of new matches is determined by a meeting function, denoted as $m(u, v)$, where u and v represent the number of unemployed workers currently looking for a job and the current number of job vacancies respectively. By assumption, the meeting function is non-negative, increasing, concave and homogeneous of degree one, so that the job-finding rate, $f(\theta) \equiv m(u, v)/u = m(1, \theta)$, is positive, increasing and concave in "market tightness" defined as the ratio of vacancies to unemployment, $\theta \equiv v/u$. Analogously, the rate at which vacancies are filled, $m(u, v)/v = f(\theta)/\theta$, is a positive, decreasing, and convex function of market tightness. Finally, matches are destroyed at the exogenous separation rate s and all agents discount future income flows at the common rate r. The matching function $m(\cdot)$, the productivity process (λ, F), and the set of parameters $\{c, r, s, z\}$ fully characterize the environment of interest.

The wage in each aggregate state, w_p, as well as the levels of unemployment and vacancies are endogenous to the model. They are determined by the match surplus sharing rule, free entry, and the law of motion for unemployment. To characterize these conditions, one needs to define the concept of match surplus.

Match surplus is the difference between the expected present value of the future incomes that the two parties to a match earn and the expected present value of income that the worker and employer forgo by being employed. Because the value of a vacancy is driven to zero by entry, match surplus is $V_p \equiv J_p + W_p - U_p$ where the values of a match to the employer, J_p, value

of the match to the worker, W_p, and the value of unemployment, U_p, are recursively defined by the continuous-time Bellman equations

$$
\begin{align}
rU_p &= z + f(\theta_p)(W_p - U_p) + \lambda(E_p U_{p'} - U_p) \tag{1}\\
rW_p &= w_p + s(U_p - W_p) + \lambda(E_p W_{p'} - W_p) \tag{2}\\
rJ_p &= p - w_p - sJ_p + \lambda(E_p J_{p'} - J_p) \tag{3}
\end{align}
$$

where E_p represents the expectation operator conditional on the current state p. By summing equations (2) and (3) and then subtracting the corresponding sides of (1), one obtains the following functional equation that the surplus value of a match must satisfy:

$$
rV_p = p - z - f(\theta_p)(W_p - U_p) - sV_p + \lambda(E_p V_{p'} - V_p). \tag{4}
$$

In all cases, these equations imply that the return on the value of the agents state is equal to the income flow obtained plus any change in value attributable to state transitions weighted by the relevant transition rate. In the unemployed worker case, the possible changes in state include a transition to employment as well as a transition to another aggregate state. Similarly, changes in values of employment and a filled job occur when the match is destroyed as well as when the aggregate state changes. Note these equations are consistent with individual rationality only if $W_p - U_p \geq 0$ and $J_p \geq 0$ for all p. As Hall (2005) emphasizes, any reasonable wage rule agreed to by the employer and worker engaged in a match must satisfy this condition.

Given that the value of not being matched is taken to be each agent's threat point, the generalized Nash solution to the bargaining problem that worker and employer face upon meeting maximizes the so-called Nash product, the geometric average of their respective shares of the match surplus, $(W_p - U_p)^\beta J_p^{1-\beta}$ where the parameter β reflects the worker's "bargaining power". The resulting sharing rule is characterized by

$$
\frac{W_p - U_p}{\beta} = V_p = \frac{J_p}{1 - \beta} \text{ if and only if } V_p \geq 0. \tag{5}
$$

It is usual to suppose that wages are renegotiated in each subsequent aggregate state so as to maintain (5). Finally, the free-entry condition requires that the expected cost of posting a vacancy is equal to the expected return. That is

$$
\frac{cv}{m(u, v)} = \frac{c\theta_p}{f(\theta_p)} = J_p. \tag{6}
$$

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An equilibrium solution to the model is a vector of functions $(w_p, \theta_p, U_p, W_p, J_p, V_p)$, all defined on the set of possible values of productivity P, that satisfy equations (1)-(6). As a contribution to Shimer's analysis, we prove that a unique equilibrium exists and that all the functions increase with productivity given reasonable technical restrictions on the matching function.

Proposition 1 *If (i) p' is stochastically increasing in p and (ii) $\theta/f(\theta)$ is a strictly increasing and concave function of θ such that $\lim_{\theta \to 0}\{\theta/f(\theta) = 0$, then a unique equilibrium exists with the property that the equilibrium functions $(w_p, \theta_p, U_p, W_p, J_p, V_p)$ are all strictly increasing in p.*

Proof. See the Appendix. ∎

Remark 2 *The conditions of (ii) are all satisfied in the case of a Cobb-Douglas matching function.*

The explicit equilibrium wage rule can easily be derived in the usual way by noting that equations (1),(2),(3), and (5) imply

$$
\begin{aligned}
& (1 - \beta)(r + s + \lambda)(W_p - U_p) \\
= {}& (1 - \beta)\left(w_p - z - f(\theta_p)(W_p - U_p) + \lambda E_p(W_{p'} - U_{p'})\right) \\
= {}& \beta(r + s + \lambda)J_p = \beta\left(p - w_p + \lambda E_p J_{p'}\right)
\end{aligned}
$$

under the assumption that the wage is renegotiated after every aggregate shock. As equation (5) holds for all p', $(1 - \beta)E_p(W_{p'} - U_{p'}) = \beta E_p J_{p'}$. This fact and the free-entry condition (6) imply that the wage outcome function takes the form

$$
w_p = \beta p + (1 - \beta)(z + \beta f(\theta_p)V_p) = \beta(p + c\theta_p) + (1 - \beta)z. \tag{7}
$$

In other words, the wage depends on the current value of aggregate productivity and increases with its realized value because current output is shared and because the value of search while unemployed is increasing in market tightness.

Under the assumption that all workers desire employment and are either employed or unemployed, the unemployment rate adjusts according to the law of motion

$$
\dot{u} = s(1 - u) - f(\theta)u
$$

where the size of the labor force in normalized at unity. Because productivity per worker is independent of employment, the unemployment rate is not an information relevant state variable. Instead, unemployment simply converges toward the state-contingent target

$$u_p = \frac{s}{s + f(\theta_p)}. \tag{8}$$

Elsewhere, Shimer (2005c) argues that the speed of adjustment, equal to the sum of the separation and job-finding rate, is large enough in practice that the negative relationship between vacancies, $v_p = \theta_p u_p$, and unemployment that it implies can be interpreted as the empirical Beveridge curve, the downward sloping relationship between vacancies and unemployment commonly observed.

3   The Elasticity of Market Tightness

Shimer's (2005a) principal claim is that the volatility of the job-finding rate and its determinant in the model, the vacancy-unemployment ratio, in US data is an order of magnitude larger than the value implied by the model for "reasonable" parameter values when fluctuations are induced by shocks to labor productivity. A critical parameter for the argument is the magnitude of the elasticity of the vacancy-unemployment ratio with respect to labor productivity. It can be derived as follows.

After substituting appropriately from the free-entry condition, equation (6), the Bellman equation implies

$$(r + s + \lambda)\frac{c\theta_p}{f(\theta_p)} + c\beta\theta_p = (1 - \beta)\left(p - z + \lambda E_p V_{p'}\right). \tag{9}$$

By taking logs and differentiating the result with respect to $\ln p$, one obtains

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \left(\frac{r + s + \lambda + \beta f(\theta_p)}{(r + s + \lambda)(1 - \eta(\theta_p)) + \beta f(\theta_p)}\right)\left(\frac{1 + \lambda\frac{\partial E_p V_{p'}}{\partial p}}{p - z + \lambda E_p V_{p'}}\right)p$$

where $\eta(\theta) = \theta f'(\theta)/f(\theta)$ is the elasticity of the job-finding rate with respect to market tightness.

At this point, Shimer (2005a) claims that the elasticity obtained when there is no aggregate shock ($\lambda = 0$) will serve as an adequate approximation for computational purposes. That is

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{r + s + \beta f(\theta_p)}{(1 - \eta(\theta_p))(r + s) + \beta f(\theta_p)} \times \frac{p}{p - z} \tag{10}$$

holds as an approximation. When evaluated at Shimer's choice of parameters, normalized median productivity $p = 1$, quarterly rates $r = 0.012$, $s = 0.10$, and $f(\theta_p) = 1.355$, labor bargaining power $\beta = 1 - \eta = 0.72$, and opportunity cost of employment $z = 0.4$, the numerical value is

$$
\begin{aligned}
\frac{\partial \ln \theta_p}{\partial \ln p} &= \frac{r + s + \beta f(\theta_p)}{(1 - \eta)(r + s) + \beta f(\theta_p)} \times \frac{p}{p - z} \\
&= \frac{0.112 + 0.72 \times 1.355}{0.72 \times 0.112 + 0.72 \times 1.355} \times \frac{1}{1 - 0.4} = 1.72.
\end{aligned}
\tag{11}
$$

However, Shimer finds that the volatility in the log of the vacancy-unemployment ratio relative to that of log productivity is over ten times larger. Namely, $\sigma_\theta / \sigma_p = 0.382/0.02 = 19.1$ given the data moments reported in Table 1 below.

Since the empirical counterpart of the derivative computed in equation (11) is in fact

$$\rho_{\theta p} \sigma_\theta / \sigma_p = 0.396 \times 19.1 = 7.56, \tag{12}$$

where σ_x represents the standard deviation of $\ln x$ and ρ_{xy} is the correlation between $\ln x$ and $\ln y$, here and in the rest of the paper, there is a question of why one should require that the elasticity of vacancy-unemployment ratio match the magnitude of the ratio of standard errors, σ_θ / σ_p, as Shimer (2005a) does. Implicitly, he is assuming that shocks to productivity are the only cause of fluctuations in vacancies and unemployment. This extreme specification, which is explicit in the simulations he performs, is not part and parcel of the original MP model. For example, fluctuations in the interest rate and the cost of posting vacancies could also be sources of volatility. Although we must admit that the model does not measure up in the sense that it fails to explain this estimate of the empirical elasticity as well, the goal post is substantially lower when one allows for these other sources of employment volatility.

Table 1: Shimer's Summary Statistics, Quarterly U.S. data, 1951-2003.

x		u	v	v/u	f	s	p
Standard Deviation σ_x		0.190	0.202	0.382	0.118	0.075	0.020
Autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation Matrix ρ_{xy}	u	1	-0.894	-0.971	-0.949	0.709	-0.408
	v	-	1	0.975	0.897	-0.684	0.364
	v/u	-	-	1	0.948	-0.715	0.396
	f	-	-	-	1	-0.574	0.396
	s	-	-	-	-	1	-0.524
	p	-	-	-	-	-	1

Source: Shimer (2005a), Table 1. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Formally, Shimer's (2005a) assertion that equation (10) serves as an adequate approximation would seem to be inconsistent with the fact that the process that he actually fits to the U.S. productivity series and uses in his simulation has a very large arrival rate, $\lambda = 4.0$ per quarter. However, the approximation also holds when arrival rate is large if the change in productivity is small. Formally, Shimer assumes that the change in "net productivity" defined as $p - z$ is determined by

$$\ln(p' - z) = \ln(p - z) \pm \Delta \text{ with probability } \frac{1}{2}\left(1 \mp \frac{\ln(p - z)}{n\Delta}\right) \qquad (13)$$

As the estimated standard deviation parameter, $\sigma = \sqrt{\lambda}\Delta = 0.0165$, $\Delta = 0.0165/2 = 0.0083$ is small given $\lambda = 4.0$. Hence, the following result justifies the use of the approximation.

Proposition 3 *Equation (10) holds in the limit as either* $\lambda \to 0$ *or* $\Delta \to 0$.

Proof. See the Appendix. ∎

3.1 Wage Rigidity

The fact that the wage is procyclical in the MP model is of little importance as a determinant of the response in the vacancy-unemployment ratio to productivity shocks given the model parameter values assigned by Shimer. Indeed, if one were to set $\beta = 0$, then the elasticity of market tightness,

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{1}{1 - \eta} \times \frac{p}{p - z} = \frac{1}{0.72} \times \frac{1}{0.6} = 2.3,$$

is still almost an order of magnitude too small, as Shimer (2005a) himself notes. As the wage is rigid in this case, $w_p = z$, from equation (7), it follows that wage flexibility *per se* is not the issue, contrary to the rhetoric of Shimer, Hall, and the much of the subsequent literature. In sum, the only important parameters determining the response of the vacancy-unemployment ratio to a productivity shock are the elasticity of the job-finding rate, η, and the opportunity cost of employment, z.

3.2   The Elasticity of the Job-Finding Rate

Shimer's value of the job-finding elasticity $\eta = 0.28$ is obtained by regressing the detrended log of his measure of the job-finding rate, derived from CPS data, on the detrended log of the ratio of vacancies as reflected in the Conference Board Help Wanted index to detrended CPS unemployment. However, the implied elasticity of the matching function with respect to unemployment, which is $1 - \eta = 0.72$, is outside the "plausible range" of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their review of the literature on matching function estimation.

Moreover, Shimer's data on vacancies and unemployment clearly imply that the Beveridge curve is close to a rectangular hyperbola. Indeed, the data moments in Table 1 imply that the OLS regression of vacancies on unemployment in his data yield the coefficient $\partial E\{\ln v | u\} / \partial \ln u = \rho_{vu} \sigma_v / \sigma_u = -0.894 \times 0.202/0.190 = -0.950$. Given the rapid adjustment of unemployment to the state-contingent target value, which is $s + f = .484$ per month, it is reasonable to argue that equation (8) should accurately represent the regression line relating the two. As this equation written as $m(u, v) = s(1 - u)$ and a linear in the logs specification of the matching function can be written as

$$\ln \mu + \eta \ln v + (1 - \eta) \ln u = \ln s + \ln(1 - u),$$

using Shimer's long run average of the unemployment rate $u = 0.0567$, it follows that

$$\frac{\partial \ln v}{\partial \ln u} = -\frac{1}{\eta} \left(\frac{u}{1 - u} + 1 - \eta \right) = -\frac{1}{\eta} \left(\frac{0.0567}{1 - 0.0567} + 1 - \eta \right) = -0.950.$$

The solution is $\eta = 0.544$, an estimate roughly equal to the lower bound on the "plausible range" of $1 - \eta$. This fact suggests that the volatility of Shimer's measure of the job-finding rate may be biased downward. But, even

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so, the elasticity of market tightness at $p = 1, z = 0.4, r = 0.012, s = 0.10,$
$f(\theta) = 1.355$, and $\beta = 1 - \eta = 0.456$

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{r + s + \beta f(\theta)}{(1 - \eta)(r + s) + \beta f(\theta)} \times \frac{p}{p - z}$$

$$= \frac{0.112 + 0.456 \times 1.355}{0.456 \times 0.112 + 0.456 \times 1.355} \times \frac{1}{1 - .4} = 1.82,$$

is still only a small fraction of either the ratio of the standard deviations
of the logs of the vacancy-unemployment ratio and labor productivity or
the empirical estimate of the elasticity of the vacancy-unemployment ratio.
Hence, disagreement regarding the magnitude of the elasticity of the job-
finding rate do not affect Shimer's conclusion given the assumed opportunity
cost of employment.

3.3   The Opportunity Cost of Employment

Shimer (2005a) sets $z = 0.4$ as a "generous estimate" of the UI replacement
ratio. Hagedorn and Manovskii (2005) argue that Shimer's choice of the
opportunity cost of employment is too low because it does not allow for the
"value of leisure" or "home production" forgone when employed as well as an
unemployment benefit. Moreover, they calibrate both the opportunity cost
of unemployment and the share parameter to match the cyclical response of
wages implied by the Solon, Barsky, and Parker (1994) wage data series and
the average profit rate computed by Basu and Fernald (1997). Their results
are $z = 0.943$ and $\beta = 0.061$. Using these numbers, the elasticity of market
tightness with respect to productivity is

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{r + s + \beta f(\theta)}{(1 - \eta)(r + s) + \beta f(\theta)} \times \frac{p}{p - z} \tag{14}$$

$$= \frac{0.112 + 0.061 \times 1.355}{0.72 \times 0.112 + 0.061 \times 1.355} \times \frac{1}{1 - 0.943} = 20.9$$

In other words, the model does explain the relative magnitudes of the vari-
ability of market tightness and labor productivity found in the data given
their parameter values even when Shimer's under estimate of η is used in the
calculation and shocks to productivity are the only source of labor-market
fluctuations.

Although the Hagedorn-Manovskii analysis does obey the letter of the law
of the model's logic, some would argue that it violates its spirit. That is, they

clearly demonstrate the estimated elasticity of the wage with respect to labor productivity, which they report as 0.45, and a small profit rate of estimated to be 3%, which they interpret as the average of $(p - w_p)/p$, require a small value of β and a large value of z. In other words, they conclude that the data are formally consistent with the model for some values of these two parameters. However, the economic implausibility of their solution is suggested by the the implication that the flow surplus enjoyed by an employed worker is miniscule. Indeed, the wage at $p = 1$ is

$$w_p = \beta(p + c\theta_p) + (1 - \beta)z = 0.061(1 + 0.213) + (1 - 0.061)0.943 = 0.959$$

given Shimer's parameters ($c = 0.213$ and $\theta_p = 1$) and the net value flow when employed is

$$\frac{w_p - z}{z} = \frac{0.959 - 0.943}{0.943} = 0.017.$$

But, do workers work for a 1.7% surplus?

Hagedorn and Manovskii respond by arguing that a value of z near p for the marginal worker is reasonable. Although that point is correct, its validity is irrelevant because job creation depends on the average value of z, not the marginal value. The proof follows.

Given heterogeneity in z, the value of unemployment for the marginal worker is equal to the value of non-participation. In other words, the value of z for the marginal participant, denote it as $\underline{z}_p$, solves $\underline{z}_p = r\underline{U}_p$ where $\underline{U}_p$ represents the marginal worker's value of unemployed search. Because

$$\begin{aligned} rU_p(z) &= z + f(\theta_p)(W_p(z) - U_p(z)) \\ rW_p(z) &= w_p(z) + s(U_p(z) - W_p(z)) \end{aligned}$$

hold as approximation for all z given Shimer's productivity process,

$$r\underline{U}_p - \underline{z}_p = f(\theta_p)(\underline{W}_p - \underline{U}_p) = \frac{f(\theta_p)\left(\underline{w}_p - \underline{z}_p\right)}{r + s + f(\theta_p)} = \frac{f(\theta_p)\beta\left(p - \underline{z}_p\right)}{r + s + f(\theta_p)} = 0$$

from the wage equation $\underline{w}_p = \beta p + (1 - \beta)(\underline{z} + \beta f(\theta_p)\underline{V}_p)$ and the fact that $\underline{V}_p = \frac{W_p - U_p}{1 - \beta} = 0$. Hence, $\underline{z}_p = p$ as they assert.

However, because the surplus value of a match depends on the worker's opportunity cost of employment, so does the surplus. Indeed, as

$$V_p(z) = \frac{p - z}{r + s + \beta f(\theta_p)}$$

holds as an approximation and the worker's type is not know when the employer posts the job, the free-entry condition is

$$\frac{cf(\theta_p)}{\theta_p} = (1 - \beta)E\{V_p | z \geq p\} = (1 - \beta)\left(\frac{p - E\{z | z \geq p\}}{r + s + \beta f(\theta_p)}\right).$$

In sum, it is the average opportunity cost of unemployed participants that matters in the determination of market tightness.

3.4   The Rigid Wage Argument Revisited

As already noted, a wage that only weakly responds to productivity shocks can account for the observed volatility in the job-finding rate but only if its level is high enough. For example, suppose that the wage w is absolutely rigid. In this case, the value of a filled job satisfies the Bellman equation

$$rJ_p = p - w - sJ_p + \lambda(E_p J_{p'} - J_p). \tag{15}$$

By the same argument used in the proof to Proposition 2,

$$\frac{c\theta_p}{f(\theta_p)} = J_p = \frac{p - w}{r + s}$$

holds as an approximation. If $(p - w)/w = 0.03$ at $p = 1$, as assumed by Manovksii and Hagedorn, then $w = 0.970$ and

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{1}{1 - \eta} \times \frac{p}{p - w} = \frac{1}{1 - 0.28} \times \frac{1}{0.03} = 46.30, \tag{16}$$

at $p = 1$. This number is well over twice that needed to explain the observed volatility of the vacancy-unemployment ratio even if productivity shocks are the only source of fluctuations.

But, the rigid wage assumption is difficult to swallow. Since aggregate shocks are common knowledge, why wouldn't negotiated wages reflect the fact that the worker's outside search option is procyclical as the Nash bargaining solution implies? Hall (2005) argues, as many others have done in the past, that the solution to bilateral monopoly problem is simply indeterminate. Any solution in the bargaining set should be regarded as a legitimate equilibrium according to Hall. Furthermore, under these circumstances it is reasonable to suppose that the wage set in previous bargains with other

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workers will serve as either a "norm" or a "focal point" for the outcome of any current bargain in every state for which this solution is jointly rational, that is when both $W_p - U_p \geq 0$ and $J_p \geq 0$ hold. He then proceeds to show that the shocks to aggregate productivity required to explain the volatility of unemployment are so small that this condition is always satisfied in simulations.

It should be point out that Hall's result is partly the consequence of his outsized estimate of the elasticity of the job-finding rate, $\eta = 0.765$, and his choice of the very high wage, $w = 0.966$. Given these number, the elasticity of market tightness at the median state $p = 1$ is

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{1}{1 - \eta} \times \frac{p}{p - w} = \frac{1}{1 - 0.765} \times \frac{1}{1 - 0.966} = 124.1$$

at $p = 1$. As a consequence, the range of possible values of p that he sets to match the observed volatility in the unemployment rate is so small that there is no problem satisfying the individual rationality conditions, $W_p - U_p \geq 0$ and $J_p \geq 0$, for all realization of p in his simulation.

4   Strategic Bargaining

4.1   The Modified Model

Following Binmore, Rubinstein and Wolinsky (1986), Hall and Milgrom (2005) point out that unemployed search is not a plausible threat in a strategic formulation of the bargaining game that a worker and employer play when they meet. If delay is the only outside option, then the wage agreement will not reflect the value of unemployment and, consequently, is more rigid than that implied by the standard sharing rule.

Specifically, if the worker receives the unemployment benefit z while negotiating, it represents the flow value of delay. Suppose that the employer bears no cost of delay. In this case, Hall and Milgrom show that the unique outcome of an alternating offer strategic bargaining game in which delay is the only credible threat is

$$W_p = \frac{z}{r} + \beta \left(W_p + J_p - \frac{z}{r} \right)$$

provided that $W_p + J_p \geq U_p$ holds for all p, which they assume.[1] In other words, the present value of the opportunity cost of employment replaces the value of unemployment as the threat point.

As equations (2) and (3) imply

$$r(W_p + J_p) = p - s(W_p - U_p + J_p) + \lambda \left[E_p\{W_{p'} + J_{p'}\} - W_p - J_p \right],$$

the present value of a worker's future income given employment is

$$W_p = \frac{z}{r} + \beta \left(\frac{p + \lambda E_p\{W_{p'} + J_{p'}\} + sU_p}{r + s + \lambda} - \frac{z}{r} \right). \tag{17}$$

There are three basic components of the joint value shared by the parties to the match. The first is the contribution attributable to the current match for as long as the current aggregate state prevails. The second is the expected joint value after a change in the aggregate state weighted by the relative likelihood that the aggregate state will change before the match ends. Finally, the residual is the worker future expected income at the end of the current match weighted by the complementary likelihood that the match will end before a change in aggregate state.

Note that the first and last term increase with current productivity while the second term increases with p to the extent that current and future productivity are correlated. In contrast, the rent sharing rule in the original model, equation (5), implies that

$$W_p = U_p + \beta \left(\frac{p + \lambda E_p\{W_{p'} + J_{p'}\} + sU_p}{r + s + \lambda} - U_p \right).$$

Because U_p represents the "threat point" in the original model, the contribution of an increase in the current value of productivity through its effect on the value of unemployment state is larger.

The importance of this difference is revealed by comparing the free-entry condition in the amended model,

$$\frac{c\theta_p}{f(\theta_p)} = J_p = (1 - \beta) \left(\frac{p + \lambda E_p\{W_{p'} + J_{p'}\} + sU_p}{r + s + \lambda} - \frac{z}{r} \right).$$

---

[1] Because the game considered by Hall and Milgrom is one in which the parties alternate offer, the bargaining positions are symettric and $\beta = 1/\dot{2}$. If one reinterprets the game as one in which nature determines who will make the offer in each round where β is the probability that the worker will be picked, then (17) holds. This form makes the results more easily comparable with the models considered above.

with that of the original

$$\frac{c\theta_p}{f(\theta_p)} = J_p = (1-\beta)\left(\frac{p + \lambda E_p\{W_{p'} + J_{p'}\} + sU_p}{r+s+\lambda} - U_p\right).$$

Since the coefficient on the current value of productivity is small given Shimer's parameters, $(1-\beta)/(r+s+\lambda) = 0.5/(0.012+0.1+4) = 0.1216$ when $\beta = 1/2$ as the authors assume, and any increase in the value of unemployment has a large negative effect on the incentive to create jobs, equilibrium market tightness increases with a productivity shock only if current and future productivity are highly correlated. Conversely, because her employer shares in a worker's future fortune in the amended model, job creation is more procyclic.

4.2   The Elasticity of Market Tightness

Although the wage obtained is less procyclical, the effect on the elasticity of market tightness is still small given Shimer's parameter values. To compute the elasticity of market tightness, we use the approximations

$$W = \frac{z}{r} + \beta\left(\frac{p+sU}{r+s} - \frac{z}{r}\right) = \frac{z}{r} + \beta\left(\frac{p-z}{r+s} + \frac{s}{r+s}(U - \frac{z}{r})\right)$$

and

$$\frac{c\theta}{f(\theta)} = J = (1-\beta)\left(\frac{p+sU}{r+s} - \frac{z}{r}\right) = (1-\beta)\left(\frac{p-z}{r+s} + \frac{s}{r+s}(U - \frac{z}{r})\right).$$

Because

$$U - \frac{z}{r} = \frac{f(\theta)}{r+f(\theta)}(W_p - \frac{z}{r}) = \frac{f(\theta)}{r+f(\theta)}\beta\left(\frac{p+sU}{r+s} - \frac{z}{r}\right) = \frac{f(\theta)}{r+f(\theta)}\frac{\beta}{1-\beta}\frac{c\theta}{f(\theta)},$$

the following equation relates market tightness and productivity:

$$\left(1 - \beta\left(\frac{f(\theta)}{r+f(\theta)}\right)\left(\frac{s}{r+s}\right)\right)\frac{c\theta}{f(\theta)} = (1-\beta)\left(\frac{p-z}{r+s}\right). \qquad (18)$$

As Shimer's parameter values imply that $r/f(\theta_p) = 0.012/1.355 = 8.86 \times 10^{-3}$ is a tiny number,

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{1}{1-\eta} \times \frac{p}{p-z} = \frac{1}{1-0.28} \times \frac{1}{0.6} = 2.3$$

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holds as a very good approximation. Recall, this result is precisely that obtained in the original model when the worker's share of match surplus is zero. In sum, substituting the Hall and Milgrom strategic bargaining solution for the wage rule assumed in the original model does not solve the problem, unless the opportunity cost of employment is sufficiently large.

Still, the point made by Hall and Milgrom is valid. Below we show that in combination with other neglected factors, it can made a substantial difference in the calculation of the elasticity of market tightness.

5   Other Neglected Factors

5.1   Capital Costs

As we have documented, the model is consistent with the data if the difference between productivity and the opportunity cost of employment is small. Why is this difference so large in the MP model? One reason is the assumption that labor in the only input. If there are complementary inputs, then net productivity less the opportunity cost of employment is the relevant difference.

Suppose that there is a capital requirement that costs k units of output. In this case, the flow of quasi-rent is $p - k - w$ which implies that the free-entry condition implied by the Hall-Milgrom bargaining solution can be written as

$$\left(1 - \beta \left(\frac{f(\theta)}{r + f(\theta)}\right) \left(\frac{s}{r + s}\right)\right) \frac{c\theta}{f(\theta)} = (1 - \beta) \left(\frac{p - k - z}{r + s}\right) \quad (19)$$

Let k equal the capital share of output in the US, about 0.35, the elasticity of market tightness is approximately

$$\frac{\partial \ln \theta_p}{\partial \ln p} = \frac{1}{1 - \eta} \times \frac{p}{p - k - z} = \frac{1}{1 - 0.28} \times \frac{1}{1 - 0.35 - 0.4} = 5.56$$

given Shimer's parameter values. In other words, the elasticity of the vacancy-unemployment rate is more than three times $(5.56/1.72 = 3.23)$ that obtained by Shimer.

5.2   Countercyclical Job Destruction

Shimer's calculations also ignore the fact that the rate of transition to unemployment, s in the model, is negatively correlated with productivity in his

data. This fact, which is implied by the Mortensen and Pissarides (1994) model with endogenous shocks, serves as an importance source of volatility in the model estimated and analyzed by Yashiv (2005). It is important here as well.

Countercyclical movements in the job destruction rate amplify the effects of productivity shocks on market tightness because the effective rate at which future profits are discounted falls in the boom and rises in the bust. In Shimer's data, the correlation between the logs of s and p is -0.524. >From equation (19) and the fact that $f(\theta)/(r + f(\theta)) \approx 1$, it follows that

$$(1 - \eta) \ln \theta = -\ln \left(\frac{c}{1 - \beta} \right) + \ln (p - k - z) - \ln (r + (1 - \beta)s).$$

Given Shimer's parameter values, setting $k = 0.35$, letting $\beta = 0.5$ as Hall and Milgrom assume, and using the OLS estimate

$$\frac{\partial \ln s_p}{\partial \ln p} = \frac{\rho_{sp} \sigma_s}{\sigma_p} = -\frac{0.524 \times 0.075}{0.020} = -1.965$$

implied by Shimer's data (See Table 1), the implied value of the elasticity at $p = 1$ is

$$
\begin{aligned}
\frac{\partial \ln \theta}{\partial \ln p} &= \left(\frac{1}{1 - \eta} \right) \left(\frac{p}{p - k - z} - \frac{(1 - \beta)s}{r + (1 - \beta)s} \frac{\partial \ln s}{\partial \ln p} \right) \\
&= \left(\frac{1}{1 - 0.28} \right) \left(\frac{1}{1 - .35 - .4} + \frac{0.5 \times 0.1}{0.012 + 0.5 \times 0.1} 1.965 \right) = 7.75
\end{aligned}
$$

The implied value of the elasticity of the vacancy-unemployment ratio with respect to labor productivity is 41% (7.75/19.1= 0.406) of the volatility in the vacancy-unemployment ratio relative to that of productivity under the assumption that productivity shocks are the only source of labor-market fluctuation.

6   Job-to-Job Flows

In the model considered by Shimer (2005a), there are no workers who flow from one job to another without an intervening spell of unemployment. In actuality, employed workers represent close to two thirds of those hired in any period. (See Nagypál (2004) among others.) In this section, we show

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that the basic model extended to include job-to-job flows explains close to half of the volatility of unemployment and vacancies when capital costs and countercyclical involuntary separations are also taken into account. However, there is empirical evidence that puts even this claim in doubt.

6.1   Search on the Job

Suppose that all jobs pay the same wage but that the amenity value of a job to the worker varies idiosyncratically across matches. Specifically, let the flow value of a job to a specific worker equal $w + x$ where w is the common wage paid in all jobs and x is a random variable representing an idiosyncratic taste component to worker compensation characterized by the c.d.f. $F(x)$. By assumption, x is iid across matches. This form of heterogeneity will induce worker movements from matches with lower to higher values of x.[2]

In order to clearly illustrate the differences between the standard model and this simple perturbation, we initially assume that workers generate job offers at rates that are independent of employment status. Because the measure of searching workers is equal to the labor force in this case, the rate at which workers, employed or not, meet jobs is simply a function of the number of vacancies. That is the measure of searching workers entering the match function is equal to the labor force, which implies that the rate at which any worker meets a vacancy is given by

$$f(\theta) = m(1, v) = m(1, \theta) \tag{20}$$

where now 'market tightness' is represented by vacancies alone,

$$\theta = v. \tag{21}$$

The value of a job with flow value of amenities equal to x in aggregate state p, denoted $W_p(x)$, solves

$$
\begin{aligned}
rW_p(x) \;=\;& w_p + x + f(\theta_p) \int \left(\max \langle W_p(z), W_p(x) \rangle - W_p(x) \right) dF(z) \\
&+ s\left(U_p - W_p(x) \right) + \lambda \left(EW_{p'}(x) - W_p(x) \right).
\end{aligned}
$$

---

[2]Alternatively, one can suppose that match product is $p + x$ which can be shared by worker and employer. Although the analysis is a bit more complicated, the quantitative results reported in the section continue to hold.

Note that the second term reflects the fact that an employed worker moves only to a job with a higher value. As the solution is monotone increasing in x for any value of p, the initial job acceptance strategy satisfies the reservation property. As the value of unemployed search solves

$$U_p = z + f(\theta_p) \int \left(\max \langle W_p(z), U_p \rangle - U_p \right) dF(z) + \lambda \left(EU_{p'} - U_p \right),$$

the reservation value is the unique solution to

$$\widehat{x}_p = z - w_p + \lambda \left(EU_{p'} - EW_{p'}(\widehat{x}_p) \right). \tag{22}$$

As is well known in search-on-the job models, the distribution of employed workers over any job characteristic generally differs from the sampling distribution as a consequence of selection. Specifically, because employed workers only move to jobs with higher values of x, and accept only jobs above the reservation value $\widehat{x}$, the aggregate state-contingent fraction of workers employed workers in jobs with idiosyncratic component less than or equal to x, denoted as $G_p(x)$, and the measure of unemployment, represented as u_p, satisfy the following steady-state conditions:

$$(s_p + f(\theta_p)(1 - F(x)) \, G_p(x)(1 - u_p) = f(\theta_p)[F(x) - F(\widehat{x}_p)]u_p$$

$$s_p(1 - u_p) = f(\theta_p)[1 - F(\widehat{x}_p)]u_p.$$

By using the second equation to eliminate u_p in the first, one obtains

$$G(x) = \frac{s[F(x) - F(\widehat{x})]}{[1 - F(\widehat{x})] \, (s + f(\theta)(1 - F(x)))} \tag{23}$$

where

$$u = \frac{s_p}{s + f(\theta)[1 - F(\widehat{x})]}. \tag{24}$$

Bargaining over a match's value is problematic when workers search on the job, particularly if the worker's idiosyncratic component of its value is not observable.[3] One simple alternative is to suppose that commitment is not possible so that bargaining takes place continuously over the division of net match product, $p - k$. Since the unemployment benefit z accrues

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[3]For an exellent discussion and analysis of the problem, see Shimer (2005b).

during the bargaining process, the Nash solution to this bargaining problems is described by the simple rent sharing rule

$$w_p = z + \beta(p - k - z).^4 \tag{25}$$

where β again represents the worker's "bargaining power."

As the worker quits a match with idiosyncratic component x at rate $f(\theta_p)[1 - F(x)]$, the value of a filled job solves

$$rJ_p(x) = p - k - w_p - (s_p + f(\theta_p)[1 - F(x)]) J_p + \lambda[E_p J_{p'} - J_p]$$

Given the wage rule, equation (25),

$$J(x) = \frac{(1 - \beta)(p - k - z)}{r + s + f(\theta)[1 - F(x)]} \tag{26}$$

holds as an approximation. In other words, the employer's discount rate of includes the quit rate.

As unemployed workers accepts any job with value of x above the reservation value and employed workers accept an alternative job when it yields a higher value of x, the probability that a job characterized by x will be accepted if $A(x) = 0$ if $x < \hat{x}$ and

$$A(x) = u + (1 - u) G(x) = \frac{s}{s + f(\theta)(1 - F(x))} \text{ if } x \geq \hat{x} \tag{27}$$

where the second equality is obtained after using equations (23) and (24) to eliminate u and $G(x)$. Free entry requires that the expect cost and return to job creation are equal. That is

$$
\begin{aligned}
\frac{c\theta}{f(\theta)} &= J_0 = \int_{\hat{x}}^{\overline{x}} A(x) J(x) dF(x) \tag{28} \\
&= \int_{\hat{x}}^{\overline{x}} \frac{(1 - \beta)(p - k - z)s}{[r + s + f(\theta)(1 - F(x))][s + f(\theta)(1 - F(x))]} dF(x) \\
&= \int_{F(\hat{x})}^{1} \frac{(1 - \beta)(p - k - z)s}{[r + s + f(\theta)(1 - y)][s + f(\theta)(1 - y)]} dy \\
&= (1 - \beta)(p - k - z)\frac{s}{rf} \left(\ln\left(\frac{r + s}{s}\right) - \ln\left(\frac{r + s + f(\theta)(1 - F(\hat{x}))}{s + f(\theta)(1 - F(\hat{x}))}\right) \right).
\end{aligned}
$$

---

[4] This is the case of capital can be rented. Otherwise a holdup problem exists and the cost of capital does not appear in this expression.

6.2 Quantitative Implications

As employment is procyclical by definition, "market tightness" is less volatile to the extent that employed workers search on the job. In the extreme case in which the employed workers contact vacancies at the same rate as unemployed workers, market tightness is simply proportional to vacancies rather than the vacancy-unemployment ratio. Using the Shimer data moments reported in Table 1, the ratio of the standard deviation of log market tightness, properly measured, to that of log productivity,

$$\frac{\sigma_\theta}{\sigma_p} = \frac{\sigma_v}{\sigma_p} = \frac{0.202}{0.020} = 10.1, \tag{29}$$

is roughly half of that implied by the model without search on-the-job.

Furthermore, the elasticity of the job-finding rate with respect to market tightness is larger than that implied by the original model and Shimer's data moments for the same reason.[5] Indeed, if all jobs are acceptable to the unemployed in all aggregate market states, an assumption which is consistent with much of the empirical evidence (Devine and Keifer (1991)), the implied elasticity of the job-finding rate with respect to market tightness is

$$\eta = \frac{\partial \ln f / \partial \ln p}{\partial \ln \theta / \partial \ln p} = \frac{\rho_{fp}\sigma_f/\sigma_p}{\rho_{vp}\sigma_v/\sigma_p} = \frac{0.396 \times 0.118}{0.364 \times 0.202} = 0.635 \tag{30}$$

rather than Shimer's value of 0.280.

Because the interest rate r is small relative to both the separation rate s and the job-finding rate f, the right hand side of the free-entry condition (28) can be approximated by its limit as r tends to zero. By L'Hopital's rule

$$\frac{s}{f} \lim_{r \to =} \left(\frac{\ln\left(\frac{r+s}{s}\right) - \ln\left(\frac{r+s+f(1-F(\widehat{x}))}{s+f(1-F(\widehat{x}))}\right)}{r} \right) = \frac{s}{f} \lim_{r \to 0} \left(\frac{\frac{1}{r+s} - \frac{1}{r+s+f(1-F(\widehat{x}))}}{1} \right)$$

$$= \frac{1 - F(\widehat{x})}{s + f[1 - F(\widehat{x})]}.$$

---

[5]This alternative specification of market tightness implies that the job-finding rate is a function of vacancies rather than the vacancy to unemployment ratio. Since the correlations of the job-finding rate with each of the two variables are roughly equal (See Table 1.), the evidence does not distinguish between the two hypotheses.

This fact is useful for computing an estimate of the elasticity. For this reason and because almost all jobs are acceptable to the unemployed ($F(\widehat{x}) \approx 0$) in practice,

$$
\begin{aligned}
(1-\eta)\frac{\partial \ln \theta}{\partial \ln p} &= \frac{p}{p-k-z} - \frac{s\frac{\partial \ln s}{\partial \ln p} + f\frac{\partial \ln f}{\partial \ln p}}{s+f} \\
&= \frac{p}{p-k-z} - \frac{s\frac{\partial \ln s}{\partial \ln p} + f\eta\frac{\partial \ln \theta}{\partial \ln p}}{s+f}
\end{aligned}
$$

holds as an approximation. Finally, solving for the elasticity and substituting in the parameter values already introduced, we obtain

$$
\frac{\partial \ln \theta}{\partial \ln p} = \frac{\frac{p}{p-k-z} - \frac{s}{s+f}\frac{\partial \ln s}{\partial \ln p}}{1 - \eta + \frac{f\eta}{s+f}} = \frac{\frac{1}{1-0.35-0.4} + \frac{0.1}{0.1+1.355} \times 1.965}{1 - 0.635 + \frac{1.355 \times 0.635}{0.1+1.355}} = 4.32.
$$

Although the relative volatility of market tightness to be explained is lower when workers search on the job, the extended model still accounts no more than 43% ($4.32/10.1 = 0.427\,72$) of the σ_θ/σ_p ratio. There are two reasons: First, the acceptance rate falls with the job-finding rate (see equation (27)) because the more picky employed become a larger share of the applicant flow as Nagypál (2005a) has pointed out. Second, the value of filling a job falls with the job-finding rate (see equation (26)) through its positive effect on the quit rate.

The assumption that employed and unemployed workers contact jobs at the same rate is also crucial for the result. In its defense, Nagypál (2005b) has shown that the relative rate of contact must be approximately unity to account for the magnitudes of the job-to-job flows inferred from CPS data given any model in which workers move from jobs of lower to jobs of higher value with the value of a given job stable over time. Still, as she also argues in the same paper, the assumption is at odds with self-reported search activity by employed workers and indirect estimates of the employed worker contract rate derived from job duration data.

7   Conclusion

Shimer (2005a) argues that the Mortensen-Pissarides equilibrium search model of unemployment explains only about 10% of the response in the job-finding

rate to an aggregate productivity shock. Some of the recent papers inspired by his critique are reviewed and commented on here. Specifically, we suggest that the sole problem is neither the procyclicality of the wage, as Shimer (2005a) and Hall (2005) argue, nor the failure to account fully for the opportunity cost of employment, as Hagedorn and Manovskii (2005) contend. Although an amended version of the model, one that incorporates the strategic bargaining wage determination rule recommended by Hall and Milgrom (2005) and also accounts for capital costs and counter cyclic involuntary separations, does much better, it still explains only 40% of the observed volatility of the job-finding rate. Finally, we find that accounting for job-to-job flows by incorporating on-the-job search does little to improve the amended model's implications for the amplification of productivity shocks. In sum, the dilemma persists.

In this analysis, we have tried to deviate from Shimer's specification and parameter values as little as possible, primarily to facilitate comparability. However, other authors using different formulations have also challenged Shimer's results. These studies include Kennan (2005), Menzio (2004), and Yashiv (2004). Determining why these authors obtain different results is beyond the scope of this paper.

8   Appendix

8.1   Proof to Proposition 1

By substitution from equation (5) and (6), equation (4) implies that an equilibrium surplus value function is a fixed point of the map

$$(TV)_p = \Gamma^{-1}\left(\frac{p - z + \lambda E_p V_{p'}}{r + s + \lambda}\right)$$

from the set of real valued functions of p to itself where $\Gamma(V)$ is the real valued function defined by

$$\Gamma(V) \equiv V + \frac{\beta c \theta(V)}{(1 - \beta)(r + s + \lambda)}$$

and $\theta(V)$ is the function implicitly defined by the free-entry condition

$$\frac{c\theta}{f(\theta)} = \frac{c}{\mu}\theta^{1-\eta} = (1 - \beta)V.$$

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Because $\theta(V)$ is continuous, increasing, and convex and $\theta(0) = 0$ under hypothesis (ii), $\Gamma(V)$ has these same properties.

The mapping T satisfies Blackwell's sufficient conditions for a contraction. Since $\Gamma^{-1}(\cdot)$ is increasing and $E_p(V_{p'} + k) \geq E_p(V_{p'})$ for all $k \geq 0$, T is increasing. Furthermore, the convexity of $\Gamma(\cdot)$ implies that $\Gamma^{-1}(\cdot)$ is concave. Hence, the additional fact that $d\Gamma^{-1}(x)/dx = 1/\Gamma'(y) \leq 1$ for all $y = \Gamma^{-1}(x)$ implies

$$
\begin{aligned}
(T(V + k)f)_p &= \Gamma^{-1}\left(\frac{p - z + \lambda E_p(V_{p'} + k)}{r + s + \lambda}\right) = \Gamma^{-1}\left(\frac{p - z + \lambda E_p V_{p'} + \lambda k}{r + s + \lambda}\right) \\
&\leq \Gamma^{-1}\left(\frac{p - z + \lambda E_p V_{p'}}{r + s + \lambda}\right) + \frac{1}{\Gamma'(V)}\left(\frac{\lambda k}{r + s + \lambda}\right) \\
&\leq (TV)_p + \beta k
\end{aligned}
$$

for any positive constant k and $\beta = \lambda/(r + s + \lambda) < 1$.

If p' is stochastically increasing in p, condition (i) holds, then T maps the set of continuous and increasing function of p into itself. Hence, the fact that T is a contraction implies that its fixed point is increasing in p. All the other equilibrium outcomes can be expressed as increasing functions of p and V_p. Finally, the assertion that V_p is strictly increasing is implied by the fact that T transforms any increasing functions into the set of strictly increasing function.

8.2   Proof to Proposition 2

The claim is an immediate implication of equation (9) in the case of $\lambda \to 0$. As the specification in (13) implies,

$$
p' - z = (p - z)e^{\Delta},
$$

it follows that

$$
\lim_{\Delta \to 0} E_p \phi_{p'} = \phi_p
$$

for any real valued integrable function ϕ of p. The free-entry condition (4) and equation (6) imply that the Bellman equation can be written as

$$
V_p = \frac{p - z - \beta c\theta_p + \lambda E_p V_{p'}}{r + s + \lambda}.
$$

It follows that,

$$V_p = \frac{r+s}{r+s+\lambda}\left(\frac{p-z-\beta c\theta_p}{r+s}\right) \tag{31}$$
$$+\frac{\lambda}{r+s+\lambda}\lim_{\Delta\to0}E_p\left(\frac{p'-z-\beta c\theta_{p'}}{r+s}\right)$$
$$= \frac{p-z-\beta c\theta_p}{r+s}.$$

Hence, the free-entry condition can be written as

$$\frac{c\theta_p}{f(\theta_p)} = (1-\beta)V_p = \frac{(1-\beta)(p-z)-\beta c\theta_p}{r+s}$$

By differentiating this expression with respect to $\ln p$, one obtains equation (10).

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