Mobility Costs, Frictional Unemployment, and Efficiency

Peter A. Diamond

Massachusetts Institute of Technology

With imperfect job information flows, it is plausible that the distribution of job offerings becomes more attractive when there are more vacancies and more unemployed. With word-of-mouth communication, this condition is derived. Given this condition, steady-state equilibrium is not efficient, with welfare increased by the introduction of unemployment compensation even though all agents are risk neutral. In this way workers become more selective in the jobs they accept.

Moving and training costs play a significant role in job-taking decisions.[1] Even if workers were equally productive in all jobs, these costs would make it sometimes worthwhile for an unemployed worker to refuse a job offer while waiting for a more attractive offer. The rate at which workers are offered jobs with different moving costs depends on the decisions of other workers as to which jobs to refuse. This externality implies that equilibrium will not generally be efficient. With the plausible assumption that job offerings become more attractive on average when the number of available jobs increases, efficiency increases when workers are induced to pass up jobs with relatively high moving costs (by unemployment compensation, e.g.).

These results are derived in a model of steady-state search equilibrium similar to one that has been used elsewhere (see Diamond and Maskin 1979 and in press; Diamond 1980*b*, 1980*c*; Mortensen, in press).

Comments from Joel Yellin and a referee, research assistance by Drew Fudenberg, and financial support from NSF are gratefully acknowledged.

[1] For a discussion of the role of employment in mobility decisions, see Bartel (1979).

[*Journal of Political Economy*, 1981, vol 89, no. 4]

Earlier analyses have focused on decisions which affect the rate at which workers receive job offers, taking as given the distribution of the quality of job offers. Here the arrival rate of job offers is taken as given, with the quality distribution endogenous. It is assumed that there is a fixed-coefficients technology. Workers and firms are taken to be risk neutral and to face a common exogenous interest rate. Both job termination and the arrival of new job offers are modeled as Poisson processes with fixed parameters. It is assumed that the wage and sharing of moving costs are negotiated,[2] with workers and firms equally good bargainers. The steady-state unemployment level is then determined by the job-taking decisions of workers. More stringent standards for job taking raise the vacancy rate, which in turn improves the distribution of job offers. Thus there can be multiple steady-state equilibria. From any steady-state equilibrium, inducing a permanent further decrease in the moving costs which workers are willing to bear raises the present discounted value of output in the economy. The optimal unemployment compensation benefit is derived in Section VI. Presentation of these results takes as given the relationship between the unemployment rate and the quality of job offers. In Section VII, a simple model of job-information flows is presented to derive an example of the way higher job availability might improve average job-offer quality. An example with an exponential distribution of moving costs is examined in detail in Sections VIII and IX. The example results in optimal benefits in the neighborhood of 60 percent of the wage.

I. Employment

Assume that all jobs are the same, with a fixed-coefficients technology that results in a flow of output, y. All jobs are subject to a risk of termination at the constant breakup rate b. This represents exogenous factors such as a transfer of the worker or the job opportunity to a different location. We ignore the possibility of the worker and the job moving together. We also ignore endogenous reasons for job termination, such as quitting for a better job or laying off workers to hire different workers.

At initial employment, the wage is negotiated, as is the sharing of setup costs. For simplicity we shall assume complete symmetry between workers and jobs (including equal numbers) implying that the net gain from commencing production is shared equally between

[2] Negotiation is assumed to be instantaneous, ruling out one route by which the presence of other workers affects a job seeker—that of waiting while a decision is made about employing a different worker.

worker and employer. In the absence of labor disutility, capital user cost, and unemployment compensation, this implies equal sharing of the flow of output and so a wage equal to $y/2$. Workers are assumed to be risk neutral and to face a constant interest rate r. With these assumptions, it is appropriate to focus attention on the expected present discounted value of earnings less moving costs. Denote by W_E and W_U the expected discounted value of earnings less moving costs for employed and unemployed workers, respectively. With complete symmetry assumptions about both finding and losing jobs, with an infinite expected life, and with analysis restricted to the steady state, W_E and W_U will not vary over time or workers.

For an employed worker, the rate of interest times expected earnings equals the wage less the expected capital loss from job termination:[3]

$$rW_E = y/2 - b(W_E - W_U). \tag{1}$$

The next step in the analysis is to consider job taking and so the determination of W_U.

II. Job Taking

Assume that unemployed workers learn about job opportunities with an exogenous[4] arrival rate, a. We assume no search costs and no decisions of an individual which affect this arrival rate. While the productivity of all jobs is taken to be the same for all workers, jobs differ across workers in the setup costs before production begins.[5] These costs reflect moving costs when workers must relocate to take new jobs and specific training costs before production can begin. The pattern of a fixed cost followed by constant output can be viewed as an approximation to the increased output that comes with on-the-job learning. We ignore the variation among jobs in commuting costs since these represent a variation in flow benefits rather than setup costs.

Denote by $G(c,u)$ the distribution of setup costs associated with jobs an individual learns about when the unemployment rate is u. A higher unemployment rate (and higher vacancy rate) is assumed to improve the distribution of moving costs in the sense that $G_u(c,u) > 0$ where

[3] Considering a worker as an asset, eq. (1) is stated as the rate of return times the value of an asset equals the flow return plus expected capital gain. It can be derived alternatively from explicit calculation of the present value of expected returns as in Diamond (1980*b*) or as the limit of a discrete time process: $W_E = (1 + r\Delta t)^{-1}[y\Delta t/2 + b\Delta tW_U + (1 - b\Delta t)W_E]$.

[4] Below, we will allow a to vary with the unemployment rate.

[5] For a detailed analysis of individual choice with setup costs, see Loikkanen and Pursiheimo (1979).

$G(c,u)$ is positive and less than one.[6] In Sections VII and VIII we derive this assumption from specific models of information flows. We take the distribution to be constant over time for a given worker. With symmetry between workers and jobs, setup costs are assumed to be equally divided between worker and job. If an unemployed worker accepts any job with a setup cost less than c^*, then, in the absence of unemployment benefits, we can write the expected discounted value of net earnings implicitly as

$$rW_U = a\int_0^{c^*} (W_E - W_U - c/2)dG(c,u). \tag{2}$$

That is, the interest rate times the expected value of earnings equals the expected gain from job taking, less setup costs.[7]

The choice problem for the individual worker is the selection of c^* to maximize W_U. Naturally, this involves accepting any job for which the setup cost is less than the expected gain from job taking:[8]

$$c^* = 2(W_E - W_U) = \left(y + a\int_0^{c^*} cdG\right)/[r + b + aG(c^*,u)], \tag{3}$$

where (1) and (2) have been solved to give the implicit equation for c^*. Workers are more willing to bear setup costs when output is greater, job finding is more difficult, the interest rate is lower, expected job duration (b^{-1}) is longer, or the unemployment rate is lower. That is, from implicit differentiation of (3) we have $\partial c^*/\partial y > 0$, $\partial c^*/\partial a < 0$, $\partial c^*/\partial r < 0$, $\partial c^*/\partial b < 0$, and $\partial c^*/\partial u < 0$.

III. Equilibrium

In steady-state equilibrium, the aggregate rate of job finding must equal the rate of job losing. Denoting the unemployment rate by u, this gives us

$$b(1 - u) = auG(c^*,u). \tag{4}$$

That is, the job breakup rate times the proportion employed equals the job acceptance rate times the proportion unemployed. Since G_u is assumed positive, we have c^* decreasing with u. One would not generally expect to find the rate, a, at which workers learn of potential jobs

[6] To interpret this assumption one needs to compare alternative economies with different steady-state employment and vacancy rates. It is not appropriate to consider a single economy over a business cycle, since in that case a rise in unemployment is accompanied by a decline in vacancies rather than a move in the same direction.

[7] As a discrete time process, we would have $W_U = (1 + r\Delta t)^{-1}(a\Delta t\{G(c^*,u)W_E + [1 - G(c^*,u)]W_U - \int_0^{c^*} c/2dG\} + (1 - a\Delta t)W_U)$.

[8] Eq. (3) comes from joint wealth maximization of firm and worker and does not depend on the equal sharing rule.

to be independent of the unemployment and vacancy rates (which are equal by assumption). We would expect a to increase with u across steady states. Since the implications of this relationship have been explored elsewhere, we take a to be exogenous here but note the implied differences in footnotes. Similar considerations hold for b.

Since c^* decreases with u in both (3) and (4), we have the possibility of multiple steady-state equilibria, that is, multiple solutions to (3) and (4). When more jobs are available (higher u), anticipated mobility costs are lower (higher G) and individuals are more selective in the jobs they take (lower c^*). Greater selectivity by workers, in turn, raises the unemployment rate. This is shown in figure 1, where equations (3) and (4) are drawn.

We note that an individual will not accept a job with an expected return below setup costs. Thus the chosen cutoff c^* is less than $y/(r + b)$. Even if expected future setup costs are close to zero, the potential loss in forgone wages implies that the chosen cutoff c^* will never be less than $y/(r + b + a)$. If all offered jobs are accepted, the equilibrium unemployment rate is $b/(a + b)$. If none is accepted, the unemployment rate goes to one.

IV. Steady-State Output

All steady-state equilibria lie along the curve given in (4). At any point on this curve we can calculate the steady-state output level per person. There are $1 - u$ employed per person giving a gross output flow per person of $(1 - u)y$. Moving costs per person equal the rate of new job taking per person of auG times the average cost of moving, $\int_0^{c^*} cdG/G$. Thus net output, Q, satisfies

$$Q = (1 - u)y - au\int_0^{c^*} cdG. \tag{5}$$

Differentiating with respect to u, with c^* given as an implicit function of u by (4), evaluating at an equilibrium where (3) holds, and using

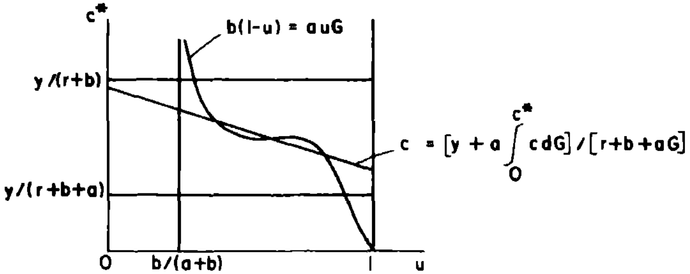

Fig. 1

integration by parts, we have

$$\begin{aligned}\frac{dQ}{du} &= -y - a\int_0^{c^*} cdG - auc^*g\,\frac{dc^*}{du} - au\int_0^{c^*} cg_u dc \\ &= -rc^* + au\int_0^{c^*} G_u dc,\end{aligned} \tag{6}$$

where $g = G_c$. The second term reflects the externality in lower moving costs of a higher unemployment rate. The first factor reflects the absence of discounting in steady-state comparisons.

V. Dynamics

The economy analyzed here cannot move directly from one steady state to another. While it is interesting to compare alternative steady states which might have occurred, proper policy analysis requires consideration of the comparative statics of the actual path of the economy. We shall analyze the effects of a policy which controls c^* directly. That is, we consider an economy where job acceptance behavior is sufficiently closely monitored to make it a government control variable. Below we will examine how unemployment compensation can be used to induce the same steady-state equilibrium without monitoring job acceptance behavior. These two modes of control correspond to stylized versions of the German and American economies.

Unemployment grows by job terminations and declines by job acceptances:

$$\dot{u} = b(1 - u) - auG(c^*,u). \tag{7}$$

Starting at a steady-state equilibrium given by (3) and (4) we shall calculate the change in the present discounted value of aggregate net output, W, from a permanent differential change in c^*, with unemployment given by (7).

That is, we want to calculate the derivative with respect to c^* of

$$W \equiv \int_0^{\infty} e^{-rt}\left\{[1 - u(t)]y - au(t)\int_0^{c^*} cdG[c;\,u(t)]\right\}dt, \tag{8}$$

where $u(t)$ satisfies (7) with an initial condition satisfying (3) and (4). Calculating this derivative (see Diamond 1980a), we have

$$\frac{\partial W}{\partial c^*} = \frac{-auc^*g}{r} + \left(\frac{y + a\int_0^{c^*} cdG + au\int_0^{c^*} cg_u dc}{r}\right)\left(\frac{aug}{r + b + aG + auG_u}\right). \tag{9}$$

Using (3) we can write this as[9]

$$\frac{\partial W}{\partial c^*} = \left(\frac{aug}{r}\right)\left[-c^* + \frac{(r+b+aG)c^* + au\int_0^{c^*} cg_u dc}{r+b+aG+auG_u}\right]$$

$$= \frac{-(a^2u^2g)\left(c^*G_u - \int_0^{c^*} cg_u dc\right)}{r(r+b+aG+auG_u)} \tag{10}$$

$$= \frac{-a^2u^2g}{r}\left(\frac{\int_0^{c^*} G_u dc}{r+b+aG+auG_u}\right) < 0.$$

Thus a government policy to move the economy out of a steady-state equilibrium in the direction of higher unemployment raises efficiency by improving the average quality of job offers.[10]

VI. Unemployment Compensation

There are several modes of intervention which will raise the equilibrium unemployment rate and so the efficiency of this economy. By increasing the importance of adjustment costs relative to the financial gains of employment, the government can induce greater selectivity in job acceptance. Thus, taxing output without allowing a deduction for adjustment costs or subsidizing unemployment or vacancies will have the desired end. Of these we shall analyze unemployment compensation. In addition to making workers more selective in accepting jobs, unemployment compensation raises the wage by one-half the unemployment compensation benefit given the negotiation assumptions we have made here.

From the perspective of the worker and employer, the net gain from production is $y - B$, where B is the unemployment benefit. With this net gain shared equally, the firm receives $0.5(y - B)$ and the worker receives a payment for forgoing unemployment compensation of B in addition to half the net gain, $0.5(y - B)$. Rewriting the two value equations (1) and (2), we have[11]

[9] If a is a function of u, (10) becomes

$$\frac{\partial W}{\partial c^*} = \frac{-au^2g}{r}\left[\frac{\int_0^{c^*}(aG_u + a'G)dc}{r+b+aG+auG_u+a'uG}\right].$$

[10] It is important to remember that we are considering policies that increase both vacancies and unemployed together. For an analysis of the gain from aggregate demand stimulation, see Diamond (1980*c*).

[11] To derive (11) formally, we need first to reconsider the equal-sharing rule above. That was derived from equations for W_E and W_U, similar equations for the values of

$$rW_E = 0.5(y + B) - b(W_E - W_U)$$
$$rW_U = B + a\int_0^{c^*} (W_E - W_U - c/2)dG(c,u). \tag{11}$$

Then, the chosen cutoff cost satisfies

$$c^* = 2(W_E - W_U) = \left(y - B + a\int_0^{c^*} cdG\right)/(r + b + aG). \tag{12}$$

Increasing the unemployment benefit by \$1.00 decreases the cutoff level of costs selected at a constant unemployment rate by $(r + b + aG)^{-1}$ dollars. The induced increase in unemployment magnifies this effect.

Setting equation (9) equal to zero, we can derive the cutoff cost level which would be optimal in a steady state. That is, an optimal trajectory will, asymptotically, have the cutoff cost level, c^*, satisfying[12]

$$c^* = \frac{y + a\int_0^{c^*} cdG + au\int_0^{c^*} cg_u dc}{r + b + aG + auG_u}. \tag{13}$$

Solving (12) and (13), we can derive the unemployment compensation benefit which holds asymptotically on the efficient trajectory (i.e., which supports the asymptotic optimum):

$$B^* = \frac{\left(y + a\int_0^{c^*} cdG\right)auG_u - au\int_0^{c^*} cg_u dc(r + b + aG)}{r + b + aG + auG_u}. \tag{14}$$

Substituting from the first-order condition for c^*, B^* can be written alternatively as[13]

$$B^* = au\left(c^*G_u - \int_0^{c^*} cg_u dc\right) = au\int_0^{c^*} G_u dc > 0. \tag{15}$$

Passing up a job offer alters the trajectory of the economy and so generates a pattern of externalities that varies over time. The present

---

filled and vacant jobs, W_F and W_V, and an equal sharing of surplus value, $W_E - W_U = W_F - W_V$. With unemployment compensation and equal sharing of setup costs the four value equations can be expressed in terms of the wage, w:

$$rW_E = w - b(W_E - W_U),$$
$$rW_F = y - w - b(W_F - W_V),$$
$$rW_U = B + a\int (W_E - W_U - c/2)dG,$$
$$rW_V = a\int (W_F - W_V - c/2)dG.$$

Combining these with the equal gain rule gives (11).

[12] From (13) and the first line of (6), we note that $dQ/du < 0$ at u^*.

[13] If a varies with u, (15) becomes $B^* = u\int_0^{c^*} (aG_u + a'G)dc$.

discounted value of the social gain from accepting a job with setup cost c can be derived by differentiating W, in (8), with respect to u. Evaluated at a steady state this gives

$$-\frac{\partial W}{\partial u} - c = \frac{y}{r + b + aG + auG_u} - c. \tag{16}$$

In an efficient equilibrium this social gain must equal the private gain $2(W_E - W_U) - c$. Equating the social and private gains, using (12), we have an alternative derivation of the asymptotically optimal unemployment compensation.

VII. Discrete Example

Having examined the implications of the dependence of the distribution of adjustment costs of job offers on the unemployment rate, we now consider an example of how word-of-mouth communication of job availability can generate such dependence.[14] In this section we consider an example with two locations. Specific training costs are c_1. There are no moving costs if the job and worker are in the same location. There are moving costs c_2 if the job and worker are in different locations. For the next section we consider an example where c is distributed exponentially with a parameter that depends linearly on the unemployment rate.

Assume that employed workers learn of a job opportunity in the same location and inform their unemployed friends of its existence. Assume that the process of job communication is such that attempted communication about any vacancy is a Poisson process with constant parameter a. (In practice it is likely that there are more attempted communications of new job vacancies than of old ones.) Each potential communicator knows n workers who could fill this job. Assume that each of the n has a probability u of being unemployed and an independent probability p of being in the same location. With probability $(1 - u)^n$ all n friends are employed and there is no one to tell of the job. If the communicator does have unemployed friends, he only tells a friend in the other location if none of his unemployed friends are in the same location. That is, he tells the friend to whom the information is most valuable. The probability of his telling someone in the other location is $[(1 - up)^n - (1 - u)^n]$—the probability of no unemployed friends with the same location less the probability of no unemployed friends. Then $1 - (1 - up)^n$ is the probability of telling a

[14] For previous analyses of word-of-mouth communication, see Boorman (1975) and Satterthwaite (1979).

friend in the same location. The greater the unemployment rate, the greater the probability of telling a friend in the same location.

With everyone following the same behavior rule, there are two candidates for equilibrium: accept only jobs at the same location, or accept any job offer. Let u_1 and u_2 be the unemployment rates under these two behavior rules. Then the equilibria satisfy

$$\begin{aligned} b(1 - u_1) &= au_1[1 - (1 - u_1 p)^n] \\ b(1 - u_2) &= au_2[1 - (1 - u_2)^n]. \end{aligned} \tag{17}$$

For some parameter values both of these equilibria will exist. Rather than pursue this example in more detail, we turn to a similar example with a continuum of locations, which can be considered to be around a circle. It is then assumed that the likelihood of knowing an individual is exponentially distributed with the distance to his location and that communication goes to the unemployed person for whom the moving costs are smallest.

VIII. Exponential Example: Steady-State Properties

We assume that c has an exponential distribution with coefficient nu. This example is chosen to fit the discussion above since the minimum of a random sample of size nu from the exponential distribution with coefficient 1 is exponential with coefficient nu. Thus we assume that

$$G(c,u) = 1 - e^{-nuc}. \tag{18}$$

With this distribution, the two equilibrium equations (3) and (4) satisfy

$$c^*(r + b + a) = y + \frac{a}{nu}(1 - e^{-nuc^*}) \tag{19}$$

$$b(1 - u) = au(1 - e^{-nuc^*}). \tag{20}$$

There exists a unique solution to this pair of equations with u between $b/(a + b)$ and one and c^* between $y/(r + b + a)$ and $y/(r + b)$.

In a steady state (whether equilibrium or not), the net output flow satisfies, from equation (5),

$$\begin{aligned} Q &= (1 - u)y - an^{-1}[1 - e^{-nuc^*}(1 + nuc^*)] \\ &= (1 - u)y - \frac{b(1 - u)}{nu} \\ &\quad - \left(\frac{a}{n}\right)\left[1 - \frac{b(1 - u)}{au}\right] \ln\left[1 - \frac{b(1 - u)}{au}\right]. \end{aligned} \tag{21}$$

The asymptotically optimal unemployment compensation satisfies, from equation (15),

$$B^* = \left(\frac{a}{nu}\right)[1 - e^{-nuc^*}(nuc^* + 1)]$$
$$= \frac{b(1-u)}{nu^2} + \left(\frac{a}{nu}\right)\left[1 - \frac{b(1-u)}{au}\right] \ln\left[1 - \frac{b(1-u)}{au}\right]. \tag{22}$$

From (21) and (22) we note that

$$Q = (1-u)y - uB^*. \tag{23}$$

Alternatively this follows from (15) since, with the exponential distribution, $\int uG_u = \int cg$. From (13) we note that at the steady-state optimum the maximal acceptable moving costs satisfy

$$\frac{y}{c^*} = \frac{b}{u^*} + r. \tag{24}$$

To find the optimal unemployment rate, we solve (20) and (24) simultaneously.

Some examples are shown in table 1. For these calculations, parameters were chosen for a, b, and r. Next an equilibrium unemployment rate in the absence of government intervention, u, was chosen. This implied a particular value for the product ny, and the remaining calculations were done for this value. In addition to steady-state comparisons, the last column of table 1 reports the percentage change in the present discounted value of net output along the trajectory from the equilibrium steady state to the optimal one where the fraction of accepted jobs, G, is held constant at its asymptotically optimal value. As detailed in the next section, the increase in net output along the trajectory exceeds the difference between steady states.

As a guide to interpreting the table, consider the fourth row. In an economy where the unemployed receive 10 job offers a year, the expected duration of a job is 5 years, the interest rate is 5 percent, and the equilibrium unemployment rate is 4 percent, 3 percent of gross output is spent on moving costs, workers are just willing to spend 34 percent of a year's output on moving, and workers accept 48 percent of jobs they hear about. The asymptotically optimal unemployment rate is 4.7 percent, which can be induced by unemployment compensation equal to 60 percent of the wage. In this equilibrium workers accept 41 percent of job offers representing an 18 percent increase in the expected duration of unemployment, and steady-state net output is higher than in the no-compensation equilibrium by 0.23 percent

TABLE 1

EXPONENTIAL EXAMPLE

a	b	r	u	$\frac{Q}{(1-u)y}$	$\frac{c^*}{y}$	$G(c^*,u)$	u^*	$\frac{B^*}{w}$	G^*	$\frac{Q^*}{Q}$	$\frac{W'}{W}$
10	.2	.05	.01965	.996	.12	.998	.01971	.31	.994	1.0000	1.0000
...	...	...	.02	.99	.13	.98	.0205	.42	.96	1.0001	1.0001
...	...	...	.03	.98	.25	.65	.0344	.59	.56	1.0013	1.0014
...	...	...	.04	.97	.34	.48	.0470	.60	.41	1.0023	1.0024
...	...	...	.05	.96	.43	.38	.0593	.61	.32	1.0032	1.0034
...	...	...	.06	.95	.52	.31	.0715	.61	.26	1.0040	1.0043
...	...	...	.07	.94	.61	.27	.0835	.60	.22	1.0048	1.0051
...	...	...	.08	.93	.69	.23	.0954	.60	.19	1.0054	1.0059
...	...	...	.09	.93	.77	.20	.1071	.59	.17	1.0061	1.0066
...	...	...	.10	.92	.85	.18	.1188	.59	.15	1.0066	1.0073
5	.2	.05	.04	.99	.27	.96	.0415	.45	.92	1.0002	1.0002
10	...	...	...	.97	.34	.48	.0470	.60	.41	1.0023	1.0024
20	...	...	...	.97	.36	.24	.0484	.62	.20	1.0031	1.0032
50	...	...	...	.96	.37	.10	.0491	.63	.08	1.0036	1.0037
100	...	...	...	.96	.37	.05	.0493	.64	.04	1.0037	1.0038
1,000	...	...	...	.96	.38	.005	.0494	.64	.004	1.0038	1.0039
10	.05	.05	.04	.97	1.40	.12	.0487	.61	.10	1.0028	1.0032
...	.1	...	...	.97	.71	.24	.0483	.62	.20	1.0029	1.0031
...	.2	...	...	.97	.34	.48	.0470	.60	.41	1.0023	1.0024
10	.2	.02	.04	.97	.35	.48	.0471	.61	.40	1.0024	1.0025

with a gain of 0.24 percent along the constant G trajectory from the initial equilibrium to the asymptotic optimum.

The examples show a surprisingly consistent pattern. When the equilibrium unemployment rate is very close to $b/(a + b)$, the minimum achievable,[15] the optimal unemployment compensation is small. As u rises, B^*/w rises very rapidly, reaching the neighborhood of 60 percent when u is about one percentage point above the achievable minimum. The ratio B^*/w stays in the neighborhood of 60 percent for all calculated values, which included values of u as integer percentages up to 10 percent. The same pattern arose for all calculated values of a, b, and r.

IX. Continuous Example Dynamics

The transition from the equilibrium steady state to the optimal one shows a larger change in net output than does the steady-state comparison. To see this, let us consider a policy of changing the proportion of jobs accepted by an unemployed worker. (We shall also consider the optimal policy below.) If workers are accepting all jobs with costs below $c^*(t)$, then the proportion of jobs accepted equals $1 - e^{-nu(t)c^*(t)}$. Let us consider the policy which holds this proportion constant over time at the level that occurs in the asymptotic steady state which we write as G^*. Then the economy follows the differential equations

$$\dot{u}(t) = b[1 - u(t)] - au(t)G^* \qquad (25)$$

$$\frac{\dot{c}^*(t)}{c^*(t)} = \frac{-\dot{u}(t)}{u(t)}.$$

We assume an initial condition at time t_0 of the steady-state equilibrium. The immediate effect of the change in policy is to decrease aggregate setup costs without changing gross output. Over time the unemployment rate rises, decreasing gross output but keeping aggregate setup costs constant. This pattern is shown in figure 2. To calculate W along this trajectory, begin by considering aggregate setup costs. These equal

$$au(t)\int_0^{c^*(t)} cnu(t)e^{-nu(t)c}dc = \frac{a}{n}\{1 - e^{-nu(t)c^*(t)}[nu(t)c^*(t) + 1]\}. \qquad (26)$$

Since $u(t)c^*(t)$ is constant, we have

$$Q(t) = Q^* + y[u^* - u(t)]. \qquad (27)$$

[15] When $a = 10$ and $b = 0.2$, $b/(a + b) = .019608$.

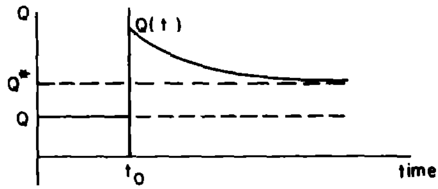

FIG. 2

Solving (25) for $u(t)$, we have

$$u(t) = u^* + (u - u^*)e^{-(bt/u^*)}. \tag{28}$$

Thus the present discounted value of output satisfies

$$W' = \int_0^\infty e^{-rt}Q(t)dt = \frac{Q^*}{r} + \frac{y(u^* - u)}{\left(r + \frac{b}{u^*}\right)}. \tag{29}$$

Since the optimal unemployment rate exceeds the equilibrium rate, the value of output along this path exceeds its value at the optimal steady state. The excess of W' over equilibrium output, W, is shown in table 1. The addition to the value of unemployment compensation from analyzing the dynamic path is small but noticeable for the examples calculated.

In addition to considering this path, which was chosen for its ease of analysis, it is interesting to analyze the optimal path assuming the government could control $c^*(t)$. The choice problem is

$$\max_{c^*(t)} \int_0^\infty e^{-rt}\left([1 - u(t)]y - \frac{a}{n}\{1 - e^{-nu(t)c^*(t)}[1 + nu(t)c^*(t)]\}\right) dt$$
$$\text{s.t. } \dot{u}(t) = [1 - u(t)]b - au(t)[1 - e^{-nu(t)c^*(t)}]. \tag{30}$$

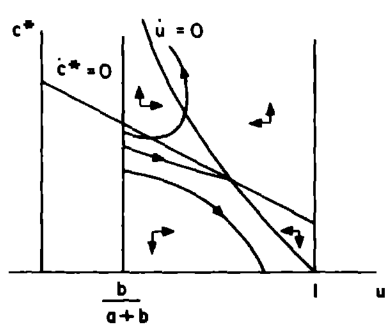

FIG. 3

The first-order condition for $\dot{c}^*$ is

$$\dot{c}^*(t) = (r + b)c^*(t) - y + ac^*(t)[1 - e^{-nu(t)c^*(t)}]. \quad (31)$$

The phase diagram for the optimal path is shown in figure 3. Along the optimal path, $c^*(t)$ is falling while $u(t)$ is rising. Their product, and so the proportion of job offers accepted, is rising. In terms of figure 2 the optimal path of net output converges to Q^* more rapidly than the path with uc^* constant, starting from a higher initial net output level.

Baily (1977) and Flemming (1978) have analyzed optimal unemployment compensation assuming risk-averse workers and no externalities in the labor allocation process. This paper takes the opposite tack of assuming risk-neutral workers and externalities. It seems likely that workers are risk averse and externalities are present, making a much stronger case for unemployment compensation.

References

Baily, Martin N. "Unemployment Insurance as Insurance for Workers." *Indus. and Labor Relations Rev.* 30, no. 4 (July 1977): 495–504.

Bartel, Ann P. "The Migration Decision: What Role Does Job Mobility Play?" *A.E.R.* 69, no. 5 (December 1979): 775–86.

Boorman, Scott A. "A Combinatorial Optimization Model for Transmission of Job Information through Contact Networks." *Bell J. Econ.* 6, no. 1 (Spring 1975): 216–49.

Diamond, Peter A. "An Alternative to Steady State Comparisons." *Econ. Letters* 5 (1980): 7–9. (*a*)

———. "Wage Determination in Search Equilibrium." Working Paper no. 253, M.I.T., Dept. Econ., January 1980. (*b*)

———. "Aggregate Demand Management in Search Equilibrium." Working Paper no. 268, M.I.T., Dept. Econ., November 1980. (*c*)

Diamond, Peter A., and Maskin, Eric. "An Equilibrium Analysis of Search and Breach of Contract. I. Steady States." *Bell J. Econ.* 10 (Spring 1979): 282–316.

———. "An Equilibrium Analysis of Search and Breach of Contract. II. A Nonsteady State Example." *J. Econ. Theory*, in press.

Flemming, John S. "Aspects of Optimal Unemployment Insurance: Search, Leisure, Savings and Capital Market Imperfections." *J. Public Econ.* 10, no. 3 (December 1978): 403–25.

Loikkanen, Heikki A., and Pursiheimo, Ulla. "On an Extended Model of Job Search." Working Paper no. 118, Univ. Helsinki, Dept. Econ., 1979.

Mortensen, Dale T. "The Matching Process as a Noncooperative Bargaining Game." In *Economics of Information and Uncertainty*, edited by John J. McCall. Chicago: Univ. Chicago Press (for Nat. Bur. Econ. Res.), in press.

Satterthwaite, Mark. "Consumer Information, Equilibrium Industry Price, and the Number of Sellers." *Bell J. Econ.* 10, no. 2 (Autumn 1979): 483–502.